George Secor's 17-Tone Well-Temperament and Neo-Medieval Music

by Margo Schulter

WHEN I LEARNED of George Secor's 17-tone well-temperament[1] in June 2001 through an article by Brian McLaren in an old archive of the Alternate Tuning List on the Internet[2], my first reaction was one of delight and awe. How did George Secor, in 1978, come up with a tuning which seemed to fit so well my own kind of "neo-medieval" style taking as its inspiration the music of Gothic Europe during the 13th and 14th centuries?

Having devised my own 17-note well-temperament the previous summer without knowledge of this precedent, I was intrigued by the similarities and contrasts between the two systems. Both tunings shared some fifths and other intervals of identical size, but woven into quite different overall designs. At this point, I knew that I had to tune Secor's system; and also, that I wanted to get in touch with its author.

When I did tune it up for the first time in early September, I was immediately taken by its beauty for usual and more "unusual" neo-medieval progressions alike. Very happily, John Chalmers had provided me with George Secor's e-mail address so that I could promptly express my appreciation for this wonderful tuning, and at the same time share some first impressions:

> Your major thirds at 428.880 cents have an exquisitely "sunny" and brilliant feeling in the kind of timbre I'm using (a bit like a portative or positive organ), lending a luminous quality to 14th-century cadences where these intervals often expand to fifths, and major sixths to octaves.
>
> The many neutral types of intervals, including your near-pure approximations of 11:6, have for me a rather mysterious and "Impressionistc" quality. They invite many variations or "altered" versions of standard medieval progressions, another delight of your tuning. [Letter to George Secor, September 5, 2001]

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[1] George Secor, "17-Tone Well Temperament," *Interval*, Volume 1, Number 1 (Spring 1978), pp. 4-5.

[2] Brian McLaren, "Secor well temperament," as posted by Gary Morrison, "Post from Brian McLaren," 13 July 1994, http://www.mills.edu/LIFE/CCM/ftp/tuning/list/archive/jul941. Here I must thank Paul Erlich for posting "An old Mills archive with Jeff Scott, Brian McLaren, and others," 18 June 2001, http://groups.yahoo.com/group/metatuning/messages/284, providing a link to this archive — and thus, for me, to Secor's well-temperament.

As it happens, seeking out Secor's "near-pure 11:6" neutral sevenths on the wrong side of the tuning circle, I had actually encountered related intervals of a hue more accurately described as "near-13:7," a distinction that would be clarified as my enthusiastic exploration continued (see Section 5.2).

Yet more happily, this was an adventure I would share with a very special guide and friend, for my letter brought a response from George Secor opening a most fruitful dialogue — or indeed, as he has suggested in a recent letter, a kind of open seminar on tunings and music.

In this first letter, Secor, designer of the Generalized Keyboard Scalatron along with Erv Wilson and Richard Harasek[3], shared a fascinating detail about his own early experiences with the 17-tone well-temperament (17-WT for short) he developed in February 1978 and published in the first issue of *Interval* that spring:

> Old music was the furthest thing from my mind when I designed the 17-WT tuning, but the idea of using it in a two-voice medieval style did in fact occur to me when I first tried it on the Scalatron. [George Secor, Letter, 10 September 2001]

What he had tried on the Scalatron in 1978 was a standard medieval progression with a major sixth to an octave, "with one big difference": the two voices moved "in equal intervals of two degrees each (neutral seconds)." He also mentioned such a variation on a standard progression from major third to fifth.

Reading this, I realized that I hadn't considered this type of progression (see Section 3.2) in my experiments in a variety of neo-medieval tunings with assorted chromatic progressions and diesis or comma shifts. Guessing correctly that it would sound quite special, I tried a three-voice version like this, with each voice moving by Secor's interval of two degrees; here I use a "Pythagorean-style" notation, only one possible choice (see Section 2), with C4 showing middle C:

C#4	Eb4
G#3	Bb3
E3	Eb3

Delighted by this "equable" resolution, a term we adapt from Harry Partch, I found that it led in many open musical directions, including some scale structures which the two of us would explore in our correspondence (Section 5), and an intriguing property of the 17-note circle which I came upon when trans-

[3]See, e.g., George Secor, "The Generalized Keyboard Scalatron," *Xenharmonikon* 3 (Spring 1975), and "Specifications of the Secor Generalized Keyboard Scalatron," *Xenharmonikon* 4 (Fall 1975); and also Erv Wilson, "The Bosanquetian 7-Rank Keyboard After Poole and Brown, *Xenharmonikon* 1 (Spring 1974), and Richard Harasek, "This Thing Called Scalatron," *Xenharmonikon* 3 (Spring 1975).

posing this type of resolution to different steps of the tuning (Section 3.3).

In this first letter, Secor also described a kind of experience we have often shared in the course of our adventure::

> I have had an entire weekend to think about your observations and questions, supposing that by this time all of the issue[s] that you raised would have become settled in my mind.... But the process of arriving at answers has led to more questions and, in turn, new perspectives on the approach you are taking. The ideas are still coming and are so numerous that I will need at least several more days to get everything sorted out... [Letter of 10 September 2001]

Over the next weeks, this creative process of "the 'cross-pollination' or 'collision' of ideas," as Secor called it in that first letter, would lead to one ramification closely following upon another: bi-step resolutions; "enharmonic" progressions with a feeling of mounting tension; Zalzal scales from the medieval Near East; augmented sixths and other neutral sevenths; suspensions; and "isoharmonic" sonorities featuring ratios of 11 and 13 (e.g. 7:9:11, 9:11:13).[4]

Together with George Secor's own companion article, this chronicle of my neo-medieval musical odyssey with his 17-WT may suggest some of the diverse possibilities of a beautiful tuning. As we both note, many of the progressions and idioms we discuss can apply to other tuning systems also, further multiplying the set of creative permutations.

In the process, I have come to a richer appreciation of my own 17-note well-temperament, even while regarding Secor's as my favorite, definitely "first among equals." In an Appendix, I compare the two systems with their sometimes mirrorlike contrasts and common elements, illustrating the flexibility of the 17-note circle and the different design philosophies one might bring to it.

Chronicling the special felicities of Secor's 17-WT in a neo-medieval setting raises one main complication: the likely unfamiliarity for many readers of medieval European music and its modern xenharmonic offshoots. Section 1, "Prelude to an Odyssey," therefore offers an overview of neo-medieval progressions and tuning systems with a purpose both of providing some general orientation, and of setting the stage for this 17-note tuning as a superb solution to the problem of achieving a circulating system combining approximate ratios of 3 and 7 in a simple diatonic scale.

From another viewpoint, elegantly developed by Secor in his companion arti-

[4]Secor explains that cross-pollination refers to "the interaction of complementary thoughts to produce something new and different from either taken separately," while "collision" refers to "ideas in conflict with one another, so that the path taken is in opposition to one of those, which may be a commonly held opinion." (Personal communication, 17 May 2002)

cle growing out of his xenharmonic experience over the last 35 years, such systems represent alternative histories, roads which might have been taken in other times and places and remain open to us in the 21st century. Often these roads may bring together cross-cultural elements such as the three-voice progressions of Gothic Europe and the *Zalzal* or related scales of the medieval Near East rich with neutral steps and intervals; 17-WT, as we shall see, is an ideal tuning for this kind of neo-medieval "fusion" style.

As a lover of medieval and Renaissance music for some 30 years, but a relative newcomer to the xenharmonic scene, I hope that this article conveys some of the creative excitement of Secor's "cross-pollination," a process exemplified in our shared adventure and apt to take shape in many yet unsuspected musical currents of the century now begun.

1. Prelude to an Odyssey: Neo-medieval tunings and commas

EMBARKING ON my exploration of Secor's 17-WT, I was coming from a background of about three years of experience with "neo-Gothic" or "neo-medieval" tunings.[5] My purpose here is not to give a comprehensive account of these tunings, but only to describe a few characteristic systems providing some context for my odyssey. As we shall see, 17-WT combines a very attractive mixture of features from these systems with admirable economy, while adding some unique features of its own.

My outlook on these tuning systems is doubtless influence by my keyboard setup: a Yahama TX-802 synthesizer with a resolution of 1024 steps to the octave, plus two standard 12-note MIDI keyboards (four octaves each), conveniently supporting tuning sets of up to 24 notes per octave, although larger sets are possible. As I quipped to George Secor, these specifications might be rather like those of the Scalatron — but with the "less desirable keyboard option," a point he could well appreciate as a designer of the Generalized Keyboard Scalatron![6] This setup might help to explain my taste for 24-note systems of a kind I'll describe shortly.

[5] While these two terms are largely synonymous, "neo-Gothic" more specifically focuses on the music of Gothic Europe, while "neo-medieval" may suggest the inclusion of other world musical traditions from the same general era, for example the Near Eastern tradition. Since the influence of Islamic civilization played a vital role in shaping the Gothic culture of Western Europe, this distinction may itself be quite relative; but the more encompassing term "neo-medieval" might be especially appropriate for music using Near Eastern scales not found in known Western European theory of the period.

[6] Letter to George Secor, 26 September 2001. The Generalized Keyboard Scalatrons have custom designs featuring 255 keys (50 keys per octave) or 294 keys (56 keys per octave), while the more basic version of the Scalatron, like my setup, has two conventional 12-note keyboards; the Scalatron has a resolution of 1/1024 octave, like the TX-802.

1.1. Standard Pythagorean tuning

One place to begin is with a 12-note Pythagorean tuning of a kind typical in 14th-century Western Europe, where 12-note keyboards were coming into vogue (e.g. the Halberstadt organ in 1365). This tuning consists of a chain of 11 fifths at a pure ratio of 2:3, or about 701.955 cents, here arranged from Eb to G#:

	113.69		294.13			611.73		815.64		996.09		
	2187:2048		32:27			729:612		6561:4096		16:9		
	C#		Eb			F#		G#		Bb		
C		D		E	F		G		A		B	C
1:1		9:8		81:64	4:3		3:2		27:16		256:243	2:1
0		203.91		407.82	498.04		701.96		905.87		1109.78	1200

This diagram shows a keyboard octave of C-C, although we might also have chosen F-F or A-A, for example. Melodically, as Mark Lindley observes, we have a pleasant contrast between whole-tones at a generously large 8:9, or about 203.91 cents, and diatonic semitones (e.g. E-F, B-C) at a compact and "incisive" 243:256 (about 90.22 cents).[7] These step sizes are very effective for simple melodies in various modes, and also for intricate textures for three or four voices.

In the 13th-14th century music of Continental Europe, and also in typical neo-medieval styles, the stable vertical or harmonic unit is a three-voice sonority with outer octave, lower fifth, and upper fourth, for example F3-C4-F4, with a ratio of 2:3:4 (a rounded 0-702-1200 cents). Around 1300, Johannes de Grocheio describes this sonority as manifesting *trina harmoniae perfectio*, the "threefold perfection of harmony," from which I have derived the English term *trine*.[8]

Another treatise of this epoch attributed to Jacobus of Liege derives this arrangement of intervals from the "natural" series of numbers 2-3-4, with the 2:3

[7] Mark Lindley, "Pythagorean Intonation and the Rise of the Triad," *Royal Musical Association Research Chronicle* 16:4–61 (1980), ISSN 0080-4460, at p. 6, and "Pythagorean Intonation," *New Grove Dictionary of Music and Musicians* 15:485-487, ed. Stanley Sadie (Washington, DC: Grove's Dictionaries of Music, 1980), ISBN 0333231112.

[8] Johannes de Grocheio's (or Grocheo's) treatise has variously been known as *Theoria*, *De musica*, or *Ars musicae*. For Latin text, see E. Rohloff, *Der Musiktraktat des Johannes de Grocheo* (Leipzig 1943), with passage on *trina harmoniae perfectio* at p. 44; for English translation, see Albert Seay, *Johannes de Grocheo Concerning Music* (Colorado College Music Press Texts/Translations 1), Colorado Springs: Colorado College Music Press, 1967, at p. 6.

fifth placed below and the 3:4 fourth above within the outer 2:4 or 1:2 octave.[9] In Pythagorean tuning, all three ratios are just or pure.

In a typical neo-medieval cadence for four voices, an unstable *quad* resolves to a complete 2:3:4 trine, or to its choicest concord of the fifth, as in these two examples, with unstable intervals resolving to stable ones by stepwise contrary motion:

E4	F4		D4	C4
D4	C4		B3	C4
B3	C4		G3	F3
G3	F3		E3	F3

(M6-8 + M3-5 + m3-1 + M2-4) (m7-5 + m3-1 + M3-5 + m3-1)

In the first cadence, the outer major sixth G3-E4 at 16:27 (~905.87 cents) expands to the octave F3-F4, while the lower major third G3-B3 at 64:81 (~407.82 cents) expands to the fifth F3-C4, and the minor third B3-D4 between the inner voices at 27:32 (~294.13 cents) contracts to the unison C4-C4. The upper major second D4-E4 at 8:9 expands to the upper fourth of the trine C4-F4.

In the second cadence, the outer minor seventh E3-D4 at 9:16 (~996.09 cents) contracts to the fifth F3-C4; the lower minor third E3-G3 contracts to the unison F3-F3, while the major third G3-B3 expands to the fifth F3-C4 and the upper minor third B3-D4 contracts to the unison C4-C4.

As these examples illustrate, Pythagorean thirds and sixths have rather complex ratios, giving them a dynamic quality nicely fitting their active and unstable role, and lending excitement to directed progressions. More generally, they exhibit what George Secor describes as a desirable contrast between simple and complex sonorities: here an unstable quad with a ratio of 64:81:96:108 (a rounded 0-408-702-906 cents) or 54:64:81:96 (0-294-702-996 cents) resolving respectively to an ideally pure 2:3:4 trine or 2:3 fifth.

In 13th-century terms, all the unstable intervals in these progressions have some degree of "compatibility": major and minor thirds are deemed *relatively* concordant but somewhat tense; while major seconds and minor sevenths, along with major sixths, are regarded as relatively tense but somewhat compatible, being described by Jacobus of Liege around 1325 as "imperfect concords." In the 14th century, major sixths are often placed on par with the milder thirds. As we'll see, acutely tense intervals such as minor seconds or major sevenths also play an important role in Gothic and neo-medieval styles.

[9] *Jacobi Leodiensis Tractatus de consonantiis musicalibus, Tractatus de intonatione tonorum, Compendium de musica*, ed. Joseph Smits van Waesberghe, Eddie Vetter, and Erik Visser (Divitiae musicae artis, A/IXa), Buren: Knuf, 1988, 88–122 at 122; the Latin text available on the World Wide Web, Thesaurus Musicarum Latinarum, Indiana University, http://theme.music.indiana.edu/tml/14th/JACCDM_TEXT.html.

We have now encountered two main themes of medieval and neo-medieval music: the use of narrow and efficient melodic semitones, about 90 cents in Pythagorean, and the vertical or harmonic contrast of active and often complex intervals such as thirds and sixths with stable fifths and fourths. These themes take on many shades of accentuation and variation in neo-medieval tuning systems.

To look at both the vertical and melodic dimensions of a cadential progression in a given tuning, here Pythagorean, we can use a diagram like the following:

E4	--	+90	--	F4
(204)				(498)
D4	--	-204	--	C4
(498,294)				(498,0)
B3	--	+90	--	C4
(906,702,408)				(1200,702,702)
G3	--	-204	--	F3

D4	--	-204	--	C4
(294)				(0)
B3	--	+90	--	C4
(702,408)				(702,702)
G3	--	-204	--	F3
(996,702,294)				(702,702,0)
E3	--	+90	--	F3

In this kind of "tree diagram," the numbers in parentheses show intervals in rounded cents between each voice and any voices above it in the diagram.[10] The signed numbers show the ascending (positive) or descending (negative) motion of each voice in rounded cents. Thus in these cadences, each voice moves by either a 204-cent whole-tone or a 90-cent diatonic semitone, here more specifically by descending whole-tones (-204) or ascending semitones (+90).

Connecting the two dimensions, we find that each directed two-voice resolution (m3-1, M3-5, M6-8, m7-5, M2-4) combines these melodic motions, one voice proceeding by a whole-tone and the other by a semitone. The total distance of expansion (M2-4, M3-5, M6-8) or contraction (m3-1, m7-5) is equal to the sum of these 204-cent and 90-cent motions, or 294 cents in all. This total distance is equal in a given tuning to the size of the minor third, formed from a whole-tone plus a diatonic semitone, in Pythagorean tuning 27:32 or about 294 cents (more precisely about 294.13 cents).

In 14th-century theory, resolutions of this type by stepwise contrary motion where one voice moves by a whole-tone and the other by a semitone are known as *closest approach* resolutions. Writers tell us that the major third "strives" to expand to the fifth, and likewise the major sixth to the octave, seeking to attain their stable goals as efficiently as possible.

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[10] For example, in the unstable sonority of the first cadence, the lowest voice at G3 forms a 906-cent major sixth G3-E4 (16:27), a 702-cent fifth G3-D4 (2:3), and a 408-cent major third G3-B3 (64:81) with the other voices considered in descending order. The voice at B3 forms a 498-cent fourth B3-E4 (3:4) and a 294-cent minor third B3-D4 (27:32) with the two voices above; the next-to-highest voice at D4 forms a 204-cent major second D4-E4 (8:9) with the highest voice. A four-voice sonority has six intervals in all. In actual four-voice writing, some of these intervals may be unisons at 1:1 or 0 cents, as shown in the resolving sonorities of either cadence.

The total expansion or contraction required for such resolutions provides a helpful measure of *cadential efficiency*, telling us how economically an unstable interval can reach its stable goal, or how closely it "approaches" this goal.

In Pythagorean tuning, the two dimensions neatly cooperate in fulfilling this ethos: vibrant and active major thirds and sixths lend an expansive impetus to cadential progressions, efficiently released in resolutions featuring melodic motions by incisive diatonic semitones and arriving at a satisfying stable concord, ideally a perfect 2:3:4 trine.

In taking Pythagorean intonation as our starting point, we have approached it freely from a neo-medieval perspective. While four-voice cadences like the above are found in medieval pieces, three-voice textures set the Gothic standard. In neo-medieval styles, both three-voice and four-voice forms are very common, as reflected in the examples to follow.[11]

Also, while closest approach resolutions define the 14th-century ideal, other types of progressions are also much favored in the 13th century, for example those in which each voice moves by a whole-tone (see Section 3.4).

However, the closest approach concept nicely introduces some of the main themes of medieval Pythagorean tuning and its neo-medieval offshoots: stable fifths and fourths, wide major intervals and narrow minor intervals which can efficiently expand or contract to stable ones, and incisive melodic semitones.[12] We now consider some of the intonational nuances these themes can take, leading to an appreciation of their superb realization in Secor's 17-WT.

1.2. A gentle temperament: the 704-cent neighborhood

While Pythagorean diatonic semitones at around 90 cents are quite compact, typical neo-medieval tuning systems feature yet more narrow and incisive semitones. In his companion article, George Secor suggests an optimal size of about 70±10 cents.

To obtain regular diatonic semitones in or around this range, we temper our fifths in the wide direction, so that they are somewhat larger than the pure 2:3 of Pythagorean tuning. From a vertical perspective, this tempering serves to make

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[11] In this article I freely mix medieval concepts like complete trinic concord and closest approach with newer ones like the cadential quad, a synthesis characteristic of neo-medieval music itself. For a more historically oriented outlook on Pythagorean intonation, see Margo Schulter, "Ugolino's 'Intelligent Organist' and the Seventeen-Note Octave, Part I: The Medieval Background," *1/1* 10, No. 3 (Fall 2000), pp. 1–15, 24; or my Pythagorean Tuning FAQ at http://www.medieval.org/emfaq/harmony/pyth.html (the Early Music FAQ Site edited by Todd McComb).

[12] Closest approach progressions follow the general pattern that major intervals tend to expand (e.g. M2-4, M3-5, M6-8), and minor ones to contract (e.g. m3-1, m7-5), as in the above examples.

major intervals wider and minor intervals narrower, thus facilitating yet more efficient closest approach progressions.

A characteristic family of neo-medieval temperaments has fifths in the general neighborhood centering around 704 cents, or about 2 cents wider than pure — about the same amount of tempering as in 12-tone equal temperament (12-ET), but in the opposite direction. We might take this neighborhood as extending from the exquisite 29-ET at about 703.45 cents (~1.49 cents wide) to an intriguing region around 704.61 cents (~2.65 cents wide) to be visited in Section 1.4.

Regular temperaments in this "704-cent neighborhood" share two families or "flavors" of thirds: regular major and minor thirds at or near ratios of 11:14 (~417.51 cents) and 11:13 (~289.21 cents); and "submajor/supraminor" thirds at around 17:21 (~365.83 cents) and 14:17 (~336.13 cents), actually diminished fourths or augmented seconds.

Let us consider a temperament with pure 11:14 major thirds, larger than their Pythagorean counterparts at 64:81 (formed from four pure fifths) by a comma of 891:896, or about 9.69 cents. Hence we temper each fifth in the wide direction by 1/4 of this comma (~2.42 cents), a size of about 704.377 cents. Here is a 12-note version of this versatile temperament, which also invites larger tuning sets such as 17 or 24 notes[13]:

								196:121				
	130.64		286.87			626.26		835.02		991.25		
	C#		Eb			F#		G#		Bb		
C		D		E	F		G		A		B	C
0		208.75		417.51	495.62		704.38		913.13		1121.88	1200
1:1				14:11								2:1

Melodically, we have whole-tones at about 208.75 cents, a bit larger than the pure Pythagorean 8:9; and diatonic semitones at about 78.12 cents, coming within the upper range of Secor's optimal zone.

To introduce the family of regular thirds and sixths, let us begin with the favorite final cadence of the 14th century, with a major sixth expanding to an octave and a major third to a fifth so as to arrive at a complete trine:

E4	--	+78	--	F4
(496)				(496)
B3	--	+78	--	C4
(913,418)				(1200,704)
G3	--	-209	--	F3

[13]If carried to 46 notes, this pure 11:14 temperament would make a circulating system almost identical to 46-ET, which has minutely smaller fifths at ~704.35 cents (~2.39 cents wide) and regular major thirds at ~417.39 cents (~0.12 cents narrow of 11:14).

(M6-8 + M3-5)

These extra-efficient resolutions require a total expansion of about 287 cents, the sum of the 209-cent and 78-cent steps or the size of the minor third (more precisely about 286.87 cents), in comparison to the 294 cents of Pythagorean tuning. We might say that the wider 418-cent major third and 913-cent major sixth, in comparison to their Pythagorean counterparts at 408 and 906 cents, more "closely approach" their goals of the fifth and octave.

Another popular three-voice cadence involves a sonority which Jacobus of Liege around 1325 calls the *quinta fissa* or "split fifth," with an outer fifth "split" by the middle voice into a major and minor third. In this example, the minor third contracts to a unison while the major third expands to a fifth[14]:

B3	--	+78	--	C4
(418)				(704)
G3	--	-209	--	F3
(704,287)				(704,0)
E3	--	+78	--	F3

(m3-1 + M3-5)

In just intonation, our pure 11:14 major third would have as its counterpart or "fifth's complement" a minor third at 28:33 (~284.45 cents), together forming a fifth at 2:3.[15] Here, however, the fifth is about 2.42 cents wide, and the minor third wider by the same amount than 28:33, placing it (~286.87 cents) about midway between this ratio and 11:13.[16]

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[14] Jacobus of Liege discusses a range of multi-voice sonorities in his monumental *Speculum musicae* or "Mirror of Music," *Jacobus Leodiensis Speculum Musicae*, ed. Roger Bragard, Corpus Scriptorum de Musica 3 (7 vols.), Rome: American Institute of Musicology (1955-1973). On the *quinta fissa*, see vol. 2, Book II, Chapter 31, pp. 80-81, describing the arrangement with the major third below and minor third above as "more agreeable"; Book II, Chapter 76, pp. 183-184, notes however that the converse arrangement is also possible, citing a 13th-century motet opening with the sonority A3-C4-E4. In a discussion of *cadentia*, the tendency of a more tense interval to seek a more stable or concordant one, Jacobus recommends the resolutions of minor third to unison and major third to fifth, vol. 4, Book IV, Chapter 50, p. 123. For more on Jacobus, see the articles in n. 10 above.

[15] George Secor suggested this term in a letter of 15 October 2001, and explained in a letter of 17 October 2001 that "fifth's complement" is by analogy to "two's complement" in computer programming.

[16] At the lower end of the 704-cent neighborhood, we find minor thirds at around 11:13 and major thirds somewhat smaller than 11:14, as in 29-ET (at around 289.65 cents and 413.79 cents). At the upper end of neighborhood, major thirds are slightly larger

Thirds at or near these rather complex ratios, like their Pythagorean counterparts, have a vibrant and active quality in many timbres adding excitement to cadential progressions where they satisfyingly resolve to stable sonorities such as 2:3:4 trines or 2:3 fifths, pure in Pythagorean or here gently tempered.

From these "accentuated Pythagorean" thirds and sixths we turn to our second family, featured in another version of the last progression:

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       Bb3 --  +131  --   B3
    (365)                (704)
       F#3 --  -209  --   E3
 (704,339)              (704,0)
       Eb3 --  +131  --   E3

           (A2-1 + d4-5)
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Here the supraminor third or augmented second Eb3-F#3, about 339.39 cents, contracts to a unison; the submajor third or diminished fourth F#3-Bb3, at an exact ratio in this tuning of 98:121 or about 364.98 cents, expands to a fifth. These thirds are quite close to 14:17 (~336.13 cents) and 17:21 (~365.83 cents), forming together with the outer fifth a sonority near 14:17:21.

Melodically, these resolutions involve striking progressions by chromatic semitones (Eb3-E3, Bb3-B3) of about 130.64 cents, in dramatic contrast to the incisive 78-cent diatonic semitones. These large semitones might be described as "2/3-tones," rather close to 13:14 (~128.30 cents).

Likewise, quite in contrast to the "superefficient" closest approach resolutions of our regular thirds requiring only 287 cents of total motion, these resolutions of our alternative thirds involve a full 339 cents of expansion or contraction, a distance equal to the size of the supraminor third.

Yet between these two very different families of intervals, I feel a charming musical kinship: with their rather complex ratios, they share what I have called an "active and passionate quality" very apt for a neo-medieval style.

Although I often call thirds in the 14:17:21 region "supraminor/subminor," George Secor remarks in a letter that in some instances they could also be perceived as neutral thirds:

> I never thought of 14:17:21 as a neutral triad, thinking of 14:17 more as a large minor third than a neutral third (probably because I first heard it in the context of Helmholtz's just diminished seventh chord, 10:12:14:17). Upon examining the interval sizes it seems that 14:17 has as much claim to being called a neutral third as does 13:16, and listening to it and the

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than 11:14, while minor thirds lean toward 28:33, as in a 704.61-cent tuning (at around 418.43 cents and 286.18 cents (see Section 1.4).

> 14:17:21 triad confirms this. The latter would then be the neutral triad having the ratio with the smallest integers, which is most interesting! [Letter, 5 November 2001]

Terms such as "supraminor/submajor," "neutral," or even "semineutral" may all fit, with subtle gradations as we move around different regions of the 704-cent neighborhood. A listener might find that "supraminor/submajor" nicely captures the quality of these thirds in 29-ET (~331.03/372.41 cents), but lean to a more "neutral" appreciation as we move toward the 704.61-cent region (~341.46/363.14 cents).

Whatever we call them, these "alternative thirds" were a very exciting "discovery" for me when I first encountered them in the early summer of 2000. They were something very different from anything I knew in the tradition of Gothic Europe, and yet seemed to fit beautifully with a 14th-century style, inviting a "reenvisioning of history."[17]

A 12-note temperament suffices to offer an ample theater for much alternative history, showcasing the main attractions of the neighborhood: the two contrasting and convivial families of complex and active thirds, and also the contrast between narrow diatonic and large chromatic semitones. Additionally, each region has its own secrets revealed only in larger tunings such as 24 notes.

Here I have chosen a temperament of special relevance to the 17-WT story, since it serves as a common ingredient in Secor's 17-WT and my own circulating 17-note scheme; both systems include a set of fifths at 704.377 cents providing some just 11:14 thirds (or 7:11 sixths).[18]

That summer and early fall I also explored some larger regular temperaments in different parts of the neighborhood, but it was only in October that I focused on what happens in a 24-note tuning as one moves toward the upper region where major thirds are slightly larger than 11:14. Here I was in for a special surprise: another family of intervals and cadences making this branch of alternative history yet more intricate and enchanting. These intervals also have a connection to our neo-medieval 17-WT saga, where they take center stage: let us consider them first in just intonation, then in tempered settings.

1.3. Streamlined ratios of 3 and 7: The comma of Archytas

[17] "Neo-Gothic tunings and temperaments: Meantone through a looking glass," 8 July 2001, Alternate Tuning List, http://groups.yahoo.com/group/tuning/message/11108. My first impressions were gleaned in a temperament at 704.61 cents.

[18] For the details of Secor's system, which uses eight such fifths in the more remote portion of the circle, see Section 2; my system simply takes the full 12-note temperament shown above with 11 such fifths (Eb-G#), adding five notes in wider fifths to close the circle. For a comparison of these systems, see the Appendix.

So far, we have focused on neo-medieval tunings and cadences where complex thirds and sixths resolve to pure or near-pure fifths and fourths, often by way of incisive diatonic semitonal motion. These dimensions of vertical contrast and melodic incisiveness harmonize in efficient "closest approach" resolutions.

However, we may also take a different approach, seeking to optimize melodic incisiveness and cadential efficiency while maximizing the simplicity or smoothness of both unstable and stable intervals.

We can ideally realize this goal with major and minor thirds at pure ratios of 7:9 (~435.08 cents) and 6:7 (~266.87 cents). These large major and small minor thirds make possible superbly efficient closest approach resolutions, while their ratios based on simple factors of 3 and 7 often lend a "streamlined" quality to textures and progressions. Other members of this streamlined family of intervals include the large major sixth at 7:12 (~933.13 cents), the large major second at 7:8 (~231.17 cents), and the small minor seventh at 4:7 (~968.83 cents). The small minor sixth at 9:14 (~764.92 cents), although less important in closest approach progressions, is a fine coloristic resource.

One measure of heightened cadential efficiency is the amount by which major intervals of this 7-based family are wider than their Pythagorean counterparts, and minor intervals narrower — for example, the 7:9 and 6:7 thirds at about 435.08/266.87 cents, vis-a-vis the 64:81 and 27:32 thirds at 407.82/294.13 cents. This difference is a comma of 63:64 (~27.26 cents).

Although this ratio is often known as the *septimal comma*, Secor felicitously proposes that it might be named the *comma of Archytas*, after the Greek mathematician and philosopher Archytas of Tarentum who favors 7-based ratios in many of his scales. Here I also use the term *Archytan comma*, by analogy with the familiar "Pythagorean comma."

To open a universe of neo-medieval just intonation at once old and new, we can place two usual 12-note Pythagorean keyboards at the distance of an Archytan comma. Here a carat (^) shows a note raised by this 63:64 comma:

	113.69		321.40			638.99		842.90		1023.35		
	243:224		2048:1701			81:56		729:448		1024:567		
	C#^		Eb^/D#			F#^		G#^		Bb^/A#		
C^		D^		E^	F^		G^		A^		B^	C^
64:63		8:7		9:7	256:189		32:21		12:7		27:14	128:63
27.26		231.17		435.08	525.31		729.22		933.13		1137.04	1227.26

	113.69		294.13			611.73		815.64		996.09		
	2187:2048		32:27			729:612		6561:4096		16:9		
	C#		Eb			F#		G#		Bb		
C		D		E	F		G		A		B	C
1:1		9:8		81:64	4:3		3:2		27:16		256:243	2:1

0	203.91	407.82	498.04	701.96	905.87	1109.78	1200

Each keyboard retains the usual medieval Pythagorean intervals, while sonorities involving pure ratios of 7 are available by mixing notes from the manuals. This arrangement involves one minor compromise: the fifth G#-D# or G#-Eb^ linking the two manuals is stretched about 3.80 cents wide, an amount of tempering comparable to that in 17-ET (~3.93 cents). As explained below, this adjustment accommodates the disparity between the Archytan comma and the slightly smaller Pythagorean comma at 524288:531441 (~23.46 cents). All other fifths remain pure, resulting in an open but essentially regular system.

To appreciate the harmonic and melodic qualities of streamlined progressions with pure ratios of 3 and 7, let us try new versions of the four-voice cadences presented in Section 1.1:

E^4 -- +63 -- F4
(231) (498)
D4 -- -204 -- C4
(498,267) (498,0)
B^3 -- +63 -- C4
(933,702,435) (1200,702,702)
G3 -- -204 -- F3

(M6-8 + M3-5 + m3-1 + M2-4)

D4 -- -204 -- C4
(267) (0)
B^3 -- +63 -- C4
(702,435) (702,702)
G3 -- -204 -- F3
(969,702,267) (702,702,0)
E^3 -- +63 -- F3

(m7-5 + m3-1 + M3-5 + m3-1)

The major sixth sonority G3-B^3-D4-E^4 has a pure ratio of 14:18:21:24 or 0-435-702-933 cents, and the minor seventh sonority E^3-G3-B^3-D4 a pure ratio of 12:14:18:21 (0-267-702-969 cents). In many harmonic timbres, the coupling of partials can give these sonorities a notably smooth quality in comparison to the more complex Pythagorean versions. This "smoothness" is one side of the overall streamlined effect.

The other side is the melodic incisiveness and cadential efficiency of these progressions. While one voice of each closest approach resolution moves by an 8:9 whole-tone, as in the Pythagorean versions, the other moves by a 27:28 semitone (~62.96 cents), much favored by Archytas as Secor discusses in his companion article. I also find this an ideal size of semitone, or cadential thirdtone, beautifully effective in a neo-medieval setting.

The total expansion or contraction required for such resolutions is equal to the sum of these steps, or to the 6:7 minor third at 267 cents, in comparison to 294 cents in Pythagorean or 287 cents in the regular temperament with 11:14 major thirds. In this variety of neo-medieval just intonation, harmonic smoothness and efficient cadential action in both dimensions most agreeably coincide.

An aspect of the intellectual as well as acoustical beauty of these progressions is the role of many superparticular ratios (*n+1:n*) as vertical and melodic intervals alike, as may be seen by using ratios rather than cents for our tree diagrams:

E^4	-- +28:27 --	F4	D4	-- -9:8 --	C4
(8:7)		(4:3)	(7:6)		(1:1)
D4	-- -9:8 --	C4	B^3	-- +28:27 --	C4
(4:3,7:6)		(4:3,1:1)	(3:2,9:7)		(3:2,3:2)
B^3	-- +28:27 --	C4	G3	-- -9:8 --	F3
(12:7,3:2,9:7)		(2:1,3:2,3:2)	(7:4,3:2,7:6)		(3:2,3:2,1:1)
G3	-- -9:8 --	F3	E^3	-- +28:27 --	F3

These pervasive superparticular or *epimore* ratios are characteristic of the tunings of Archytas, and of intricate just intonation systems generally.

Here these ratios are realized in a near-regular system with essentially the same transposibility as a 24-note Pythagorean tuning, from which it might have evolved in some parallel history, and *did* evolve in my own tuning practice.

In a 24-note Pythagorean scheme, corresponding notes on the two manuals such as Eb-Eb^/D# would be located at a distance of 12 pure fifths on the chain, placing them a 23.46-cent Pythagorean comma apart, and falling short of the Archytan comma by about 3.80 cents. This slight disparity between commas, or schisma, which we might call the *3-7 schisma*, is equal to a complex integer ratio of 33554432:33480783.[19] Stretching the fifth G#-D# or G#-Eb^ by this schisma places the two keyboards an Archytan comma apart to achieve pure 7-based ratios.

Interestingly, a regular 24-note Pythagorean tuning with the manuals a Pythagorean comma apart provides very useful approximations of these ratios, with major intervals narrower and minor intervals wider than their pure 7-based sizes by this 3.80-cent schisma. Closest approach progressions of near-7 sonorities involve a semitone wider than 27:28 by this same schisma, or about 66.76 cents, another excellent "thirdtone."[20] As it happens, this same quantity of 66.76 cents, plays a role in the saga of 17-WT (see Section 2).

Fortunately, the refinement of stretching G#-D# by the schisma to obtain just ratios of 7 is one of the more pleasant compromises of tuning: this fifth is about as impure as in 17-ET, and less so than many of the fifths in 17-WT, while some intervals of the regular Pythagorean type (e.g. the major third F#-A#/Bb^) with this wide fifth in their chains are slightly altered in the efficient and incisive

[19]Eduardo Sábat, *Principios de la Gama Dinámica* (Montevideo: Arca, 1994), terms this schisma Beta 2 (*Beta sub-dos)*, see Section 6.3, pp. 208-209; it has also been termed the *septimal schisma*.

[20]This interval, the amount by which 17 pure fifths fall short of 10 pure octaves, is almost identical in size to Busoni's "tripartite tone" of 1/18 octave or ~66.67 cents (precisely 66-2/3 cents), see Federico Busoni, *A New Esthetic of Music*, in *Three classics in the aesthetic of music: Monsieur Croche the dilettante hater, by Claude Debussy. Sketch of a new esthetic of music, by Ferruccio Busoni. Essays before a sonata, by Charles E. Ives* (New York: Dover Press, 1962), at pp. 93-95, where he advocates 36-ET.

direction. If we seek a just intonation system with pure ratios of 3 and 7 along with easy transposibility, then this is a very attractive solution.[21]

In a neo-medieval setting, the Archytan comma often spells *precision* and *choice*: here we inhabit a world filled with pure ratios of 3 and 7, and are free to choose between two famillies or "flavors" of sonorities and cadential progressions, the streamlined 7-based forms or the usual Pythagorean ones with their more complex or "beatful" quality.[22] These flavors or "moods" nicely complement each other as aspects of a common style, and may often freely alternate.[23]

1.4. Observing the Archytan Comma: A three-room cottage

Another road to ratios of 3 and 7, this time approximate rather than just, involves a regular temperament with major thirds slightly larger than 11:14. Such a tuning retains all the usual attractions of the region (Section 1.2), further adorned by the new intervals and progressions.

One might say that this road leads through a gate opened by the difference in size between the diatonic and chromatic semitones, a difference large enough to define a third type of semitone step featured in a new family of very effective cadential resolutions. The result in a 24-note tuning is a system with three main flavors or families of unstable sonorities and cadences, a kind of spacious three-room cottage.

It happens that when first exploring the 704-cent neighborhood, I got an idea for a 12-note temperament at what would serve as an ideal location for such a larger cottage — by a process of sheer serendipity.

My idea was a regular temperament with a ratio between whole-tone and diatonic semitone equal to Leonhard Euler's famous exponential quantity *e*,

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[21]. Tuning by ear, we could proceed by just intervals only without directly adjusting the fifth G#-D#. Placing the lower manual in a Pythagorean tuning (Eb-G#), we might then set D^4 on the upper keyboard to a pure 6:7 below F4 or a pure 7:4 below C5, tuning the rest of this manual in Pythagorean fashion. In Owen Jorgensen's terms no (direct) "tempering" is done, only pure tuning, see his *Tuning the Historical Temperaments by Ear* (Marquette: Northern Michigan University Press), ISBN 091861600X.

[22]One might add the qualification that the 14:18:21 sonority (0-435-702 cents) with 7:9 major third below and 6:7 minor third above seems in many timbres more strident than the Pythagorean 64:81:96 (0-408-702 cents), although in higher registers it may take on a smoother quality.

[23]In my own improvisations in this and related 24-note systems, I have noted a tendency to use regular Pythagorean intervals in faster contrapuntal passages, leaning more to 7-based intervals or a frequent alternation between flavors in slower passages. This may reflect the exigencies of the keyboard arrangement, as well as the heightened effect of pure sonorities in more sustained textures.

about 2.71828.[24] This temperament has a fifth at about 704.607 cents (~2.65 cents wide), with a whole-tone of 209.214 cents and diatonic semitone of 76.965 cents.

The 12-note version I tuned in June of 2000 immediately captured my imagination, providing what I have called a "delightful initiation" to this region. However, it was only in October that I excitedly considered what would happen in a 24-note version of this "e-based" tuning, and eagerly tried it.

Here is a keyboard diagram for this regular 24-note temperament, with an asterisk (*) showing a note raised by a diesis of about 55.28 cents, an interval defining the distance between the manuals (e.g. E-E*) and playing a premier cadential role in this enlarged system[25]:

	187.53		341.46			682.92		892.14		1046.07		
	C#*		Eb*/D#			F#*		G#*		Bb*/A#		
C*		D*		E*	F*		G*		A*		B*	C*
55.28		230.90		473.71	550.68		759.89		969.10		1178.32	1255.28

	132.25		286.18			627.64		836.86		990.79		
	C#		Eb			F#		G#		Bb		
C		D		E	F		G		A		B	C
0		209.21		418.43	495.39		704.61		913.82		1123.03	1200

The regular diatonic semitone at about 76.97 cents occupies an interesting place within Secor's optimal range of 70±10 cents: it has a size very close to the *chromatic* semitone of 1/4-comma meantone (~76.05 cents) or 31-ET (~77.42 cents) much favored as a cadential step by Darreg Ivor.[26]

This tuning, a close neighbor to the temperament with just 11:14 major thirds (fifths 704.377 cents), has these thirds at 418.43 cents, only 0.92 cents wider. Here are two progressions earlier presented in connection with that tuning, showing the families of regular and submajor/supraminor thirds, and the contrast between the 77-cent and 132-cent semitones:

[24] Easley Blackwood, *The Structure of Regular Diatonic Tunings* (Princeton, NJ: Princeton University Press, 1985) refers to this ratio between whole-tone and diatonic semitone in a regular tuning as R, and makes it a major theme of his theory.

[25] On my "delightful initiation" and the story of the 24-note version, see "The e-based tuning and metachromatic progressions: A feast of neo-Gothic flavors" (30 March 2001) http://groups.yahoo.com/group/tuning/message/20573; and also n. 16 above.

[26] Darreg, in "The Calmer Mood: 31 Tones/Octave," *Xenharmonic Bulletin* 9 (January 1978), urges performers in 31-ET to overcome "the flatness of the leading-tone" by playing such notes "1/31 of an octave higher," resulting in a small cadential semitone of 2/31 octave; text available at http://www.ixpres.com/interval/darreg/xhb9.htm.

B3	--	+77	--	C4	Bb3	--	+132	--	B3
(418)				(705)	(363)				(705)
G3	--	-209	--	F3	F#3	--	-209	--	E3
(705,286)				(705)	(705,341)				(705,0)
E3	--	+77	--	F3	Eb3	--	+132	--	E3
		(m3-1 + M3-5)					(A2-1 + d4-5)		

The submajor/supraminor thirds at 363.14/341.46 cents, although still not too far from 14:17:21, shade somewhat toward the more central zone of neutral thirds, generally deemed to include 13:16 and 32:39 (359.57/342.48 cents).

We now come to the third family, or room of our enlarged cottage: a streamlined realm where intervals and sonorities approximate ratios of 3 and 7, with superefficient resolutions featuring the 55.28-cent diesis. This diesis, defined as the distance between two notes located 12 fifths apart on the tuning chain (e.g. Eb-D#/Eb*), or the difference between the diatonic and chromatic semitones, is large enough to serve as a very effective cadential semitone in its own right.[27]

As I excitedly realized in October 2000, a chain of 15 fifths up (e.g. F3-D*4), or major-sixth-plus-diesis at about 969.10 cents, forms a virtually pure 4:7 minor seventh (only about 0.28 cents wide)! Likewise 14 fifths up (e.g. F3-G*3), or a major-second-plus-diesis, forms a near-pure 6:7 minor third at about 264.50 cents (~2.37 cents narrow). With 13 fourths up (e.g. G*3-C4), we get a fourth-less-diesis at about 440.11 cents, about 5.03 cents wide of a pure 7:9 major third.

Two four-voice cadences will showcase these and related vertical intervals, all within about 5 cents of pure, and their closest approach resolutions:

F4	--	+55	--	F*4	D*4	--	-209	--	C*4
(231)				(495)	(264)				(0)
D*4	--	-209	--	C*4	C4	--	+55	--	C*4
(495,264)				(495,0)	(705,440)				(705,705)
C4	--	+55	--	C*4	G*3	--	-209	--	F*3
(936,705,440)				(1200,705,705)	(969,705,264)				(705,705,0)
G*3	--	-209	--	F*3	F3	--	+55	--	F*3
		(M6-8 + M3-5 + m3-1 + M2-4)					(m7-5 + m3-1 + M3-5 + m3-1)		

In these resolutions one voice moves by the usual 209-cent whole-tone, and the other by a 55-cent diesis, requiring a total expansion or contraction of only

[27] This diesis in a regular tuning, equivalent to the Pythagorean comma, is equal to the difference between 12 fifths in a given tuning and 7 pure octaves; or between three major thirds and a pure octave; or between a pure octave and four minor thirds. The use of the term *diesis* may imply a disparity considerably greater than the Pythagorean comma (~23.46 cents), often in the range of about 35–70 cents.

264.50 cents, the size of the near-6:7 minor third.[28] With the 24-note keyboard layout reflected in this notation, these melodic diesis steps move from a note on one manual to the corresponding note on the other manual (e.g. the ascending steps C4-C*4 and F4-F*4).

As a cadential interval, this extra-narrow semitone or diesis may add emphasis and excitement, contrasting with the usual diatonic step of 77 cents. One might describe it as a highly compressed thirdtone or large quartertone, about 7.68 cents narrower than the 27:28 step favored by Archytas.

The Archytan comma is represented in this temperament by the difference between these two steps, about 21.79 cents, and also the difference between regular intervals such as major and minor thirds at 418.43/286.18 cents and their approximate 7-based versions at 440.11/264.50 cents. We might call this comma the "17-comma," the amount by which 17 fifths fall short of 10 pure octaves.

Such a spacious musical cottage has many musical possibilities to offer, with the Archytas comma (or here its tempered equivalent) often serving as an emblem of variety and choice. Indeed, there are choices of which I have become aware in this open 24-note tuning only as a result of exploring Secor's WT-17, specifically a set of "equable" cadences (see Section 3.2).[29]

However, in approaching ratios of 3 and 7, another kind of musical architecture has its own special possibilities, featured as only one aspect of what I might style the intonational palace of ratios realized with exquisite economy in 17-WT.

1.5. Dispersing the Archytan Comma: Prologue to 17-WT

Observing the Archytas comma in a just or tempered system can mean choice, variety, and also a degree of intricacy. For example, let us consider a typical 13th-century progression in which a sonority of G3-C4-D4, unstable but often apparently considered relatively concordant with its pure Pythagorean ratios of 6:8:9 (about 0-498-702 cents), leads to the cadential sixth sonority G3-B3-E4 resolving in the usual fashion to F3-C4-F4:

D4	E4	F4
C4	B3	C4
G3	-------	F3

[28]This compares with regular closest approach resolutions like those in the first three-voice cadence above, with a total motion equal to the size of this whole-tone plus a 77-cent diatonic semitone, or about 286.18 cents, the size of the regular minor third.

[29]In addition to the three main interval families or rooms of the cottage, there are also some smaller alcoves and passageways: for example, an augmented third or fourth-plus-diesis (e.g. Eb3-G#3 or D4-G*4) at about 550.68 cents, only about 0.64 cents narrow of a pure 11:8 (~551.32 cents), and a few remote intervals like those of meantone or 12-ET, e.g. Eb3-F#*3-Bb3-C#*4 at a rounded 0-397-705-1101 cents.

Suppose we desire a version with G3-B3-E4 at or near a 7-based ratio of 7:9:12, and the cadential semitones B3-C4 and E4-F4 at or near the 27:28 of Archytas. Here are likely solutions in the Pythagorean-based just system (Section 1.3) and the regular temperament at 704.609 cents (Section 1.4):

D4	E^4	F4		D*4	F4	F*4
C4	B^3	C4		C*4	C4	C*4
G3	-------	F3		G*3	-------	F*3

(Pythagorean-based just) (704.609-cent temperament)

In either system, the musician has both the extra freedom and the added responsibility indeed to *observe* the comma distinction in choosing the desired "mood" or flavor of cadence. The possible intricacies become yet more clear if we happen, for example, to begin this progression in the second system on the lower keyboard with G3-C4-D4:

D4	E4	E*4
C4	B3	B*3
G3	F#*3	E*3

In this 24-note system, a near-7:9:12 sonority (about 0-440-936 cents) must have its lowest note on the *upper* keyboard, calling for the lowest voice to move from G3 to F#*3, a 22-cent shift by the equivalent of the 63:64 Archytan comma. This sonority bring us, in the most incisive resolution, to E*3-B*3-E*4, lower by the same tempered version of the comma than the regular resolution F3-C4-F4.

Both the range of alternative resolutions offered by such a system[30], and the use of comma shifts (as in the last example) and relations between sonorities such as F3-C4-F4 and E*3-B*3-E*4 which might serve as "synonyms" for "the same basic cadential center," open many creative nuances.

At times, however, it would also be attractive to combine ratios of 3 and 7 in a simpler and more seamless fashion, with intervals such as the 7:9 major third, 6:7 minor third, and 4:7 minor seventh included or approximated within a *simple* diatonic scale. This option requires a considerable compromise in intonational accuracy compared to the 24-note systems we have considered, but has the charm of simplicity.

Suppose, for example, that we desire a regular tuning with a major third at a pure 7:9, an Archytan comma wider than the Pythagorean 64:81 formed by a

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[30] Here, for example, F#*3-B3-E4 might resolve not only to E*3-B*3-E*4 with melodic motions of a rounded 55 cents up and 209 cents down (+55/-209 for short), but also, in other progressions with some kind of "stepwise" motion in each voice, to F3-C4-F4 (-187/+77); to F*3-C*4-F*4 (-132/+132), an equable resolution corresponding to the 17-WT variety discussed in Section 3.2; or to F#3-C#4-F#4 (+209/-55).

chain of four pure 2:3 fifths (e.g. F-C-G-D-A). Each fifth must then be tempered wider than pure by 1/4 of this 27.26-cent comma, a "1/4-Archytan" temperament as we might call it for short, or about 6.816 cents. Such a tuning indeed optimizes the 7:9 third, but rather heavy in its tempering of the fifths and fourths, the primary stable concords in a typical neo-medieval style.

If we desired regular minor thirds at a pure 6:7, each formed from a chain of three fifths down or fourths up (e.g. E-A-D-G), then an even heavier temperament by 1/3 of the comma would be necessary — about 9.088 cents, going well beyond the amount of temperament for the fifth attested in known regular tunings of historical Europe. Obtaining pure 4:7 minor sevenths would require a yet more drastic 1/2-Archytan temperament, about 13.362 cents wide.

For neo-medieval music in many harmonic timbres, even a 1/4-Archytan temperament or 22-ET may compromise fifths and fourths quite heavily. In his companion article, George Secor discusses another consideration in dispersing the comma: the size of the regular diatonic semitone, which shrinks as the fifths grow larger.

These steps are about 56.145 cents in 1/4-Archytan, or 54.545 cents in 22-ET, rather smaller than Secor's optimal range of 70±10 cents. As special cadential semitones or dieses, such step sizes can be very attractive (Section 1.4). While I personally find steps of about 55 cents quite acceptable as usual semitones also, Secor remarks that they are situated "at the borderline of what we might perceive as either a semitone or quartertone (depending on the musical context)," a melodic ambiguity he finds less than ideal in a regular diatonic scale. This might be a matter of taste upon which seasoned microtonalists may have a range of opinions.[31]

From a neo-medieval perspective, the desire to treat fifths and fourths with *relative* gentleness in dispersing the comma suggests a paraphrase of an old adage: "That temperament tempers best which tempers least." One attractive

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[31] Both Secor and Ivor Darreg have suggested that the range around 55 cents may be ambiguous, somewhere between the realm of usual semitones (or thirdtones) and that of distinctly "microtonal" steps up to 50 cents or so. Ertugrul Inanc, a Turkish composer who takes an interest in the Near Eastern tradition, suggests that that the smallest melodic "semitones" in this tradition are at around 32:33 (~53.27 cents), which he explains may arise in a division of a neutral second at 10:11 (~165.00 cents) into 15:16 (~111.73 cents) and 32:33. (Personal communication, 14 November 2001). He associates this type of narrow semitone with cadences, where it serves as an alternative to the much-favored step of around 24:25 (~70.67 cents), but remarks that he has heard noted performers of Turkish music use it noncadentially also. Paul Erlich, an advocate of 22-ET, reports that he and his audiences seem to accept a regular semitone of 1/22 octave without any problems. In exploring this question further, we might distinguish between the concepts of *acceptable* and *optimal* sizes. Thus one favoring Secor's optimal range of around 60–80 cents might find acceptable either the 55-cent step of 22-ET on the small side, or the 90-cent step of Pythagorean or 100-cent step of 12-ET on the large side.

compromise is to temper so that fifths and near-7:9 major thirds will be impure by about the same amount, with this impurity precisely equal at 1/5-Archytan (fifths ~707.408 cents, ~5.453 cents wide; major thirds ~429.631 cents, ~5.453 cents narrow).

From Secor's melodic perspective, also, this kinder and gentler temperament of the fifth yields a diatonic semitone at a just 27:28 or about 62.96 cents, the ratio favored by Archytas.

In fact, the nearer portion of the tuning circle in Secor's 17-WT has a chain of nine fifths (Ab-B) very close to this size at about 707.220 cents (~5.265 cents wide), tempered slightly more gently than in the 1/5-Archytan tuning; and major thirds at 428.882 cents, about 6.202 cents narrow of 7:9.

As it happens, this precise fifth size was tailored in part to fit other factors in the design of the palace of ratios such as 11 and 13, with a chain of seven of these fifths, for example, producing a virtually just 6:11 neutral seventh (Section 5.2). For ratios of 3 and 7 approached from a neo-medieval viewpoint where fifths and fourths are the prime concords, it seems a very happy compromise, and indeed just about an optimal solution.

From an historical perspective, this temperament very close to 5/26-Archytan could be taken as almost the mirror image of another tuning: 1/4-comma meantone, in which each fifth is tempered in the *narrow* direction by 1/4 of the comma of Didymus (80:81, ~21.51 cents) in order to obtain pure 4:5 major thirds (~386.31 cents). This 1/4-(Didymic)-comma tuning, as we might call it to specify the comma dispered, compromises fifths by about 5.38 cents, almost exactly the same amount as in the nearer portion of 17-WT, albeit in the opposite direction.

In either 1/4-Didymic, a meantone temperament evidently quite popular in the 16th and early 17th centuries, or Secor's 5/26-Archytan portion of 17-WT, fifths are appreciably "beatful" in harmonic timbres and yet can have a quality of satisfying concord and repose. Comparing these 17-WT fifths in such a timbre with 22-ET fifths (tempered at ~7.14 cents wide, a bit more than 1/4-Archytan), I found not so surprisingly that the 17-WT fifths were considerably smoother or more "solid."[32]

Some features of this happy compromise for dispersing the Archytan comma may become clearer if we look at a keyboard diagram for this nearer Ab-B portion of nine fifths, or ten notes, in 17-WT:

[32] Similarly, in Renaissance styles of a kind very common around 1500 where pieces and phrases within them often conclude on a fifth-octave consonance which might in earlier medieval terms be described as a trine (e.g. D3-A3-D4), 1/4-Didymic meantone can make such sonorities of repose satisfying conclusive, albeit also a bit "beatful" or "lively."

		278.339				771.118		985.559		
		Eb				Ab		Bb		
C	D		E	F	G		A		B	C
0	214.441		428.882	492.780	707.220		921.661		1136.102	1200

From a melodic viewpoint, we have diatonic semitones at 63.90 cents, only about 0.94 cents larger than a just 27:28, and not too far from Secor's suggested center of the optimal range at around 70 cents. Whole-tones are 214.441 cents. contrasting very effectively with the narrow regular semitones. Chromatic semitones in this portion of the tuning (Bb-B, Eb-E, Ab-A) might be described as neutral seconds or "3/4-tones" at 150.543 cents, almost precisely 11:12. The disparity between these diatonic and chromatic semitone sizes facilitates some dramatic realizations of 14th-century and related chromatic idioms (Section 5.1).

Harmonically, in addition to our tempered fifths and near-7:9 major thirds, we have minor thirds (e.g. E-G, C-Eb) at 278.339 cents, about 11.47 cents wide of 6:7; and minor sevenths (e.g. D-C, C-Bb) at 985.559 cents, about 16.73 cents wide of 4:7. While illustrating the compromises in accuracy involved in dispersing the Archytan comma[33], these intervals can nevertheless happily serve as musical approximations for their just 7-based counterparts over a range of styles.

Thus C4-Eb4-G4 can represent a 6:7:9, or C4-Eb4-G4-Bb4 a 12:14:18:21, both in neo-medieval styles where these sonorities are typically treated as mildly unstable or "relatively concordant," and in styles of a kind discussed by George Secor where they may serve as fully consonant and conclusive.

In a regular temperament of this kind, or within this portion of 17-WT, we can realize a near-7-based version of the 13th-century progression discussed above simply by playing the usual diatonic notes:

D4	E4	F4
C4	B3	C4
G3	-------	F3

Two four-voice cadences show a variety of near-7 intervals in action:

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[33] Within this 10-note portion of 17-WT where fifths are tempered at about 5.265 cents, or 5/26-Archytan, each such fifth in an interval's tuning chain alters it about 5/26 of the way from a Pythagorean ratio (based on a chain of pure fifths) to a just 7-based ratio, a total distance measured by the full Archytan comma of 63:64 or about 27.26 cents. Thus a diatonic semitone (e.g. B-C) formed from 5 fifths up is altered by about 25/26 of the comma, leaving it only the remaining 1/26-Archytan or so wide of a just 27:28, about 1.14 cents. Major thirds (4 fifths up), minor thirds (3 fifths down), and minor sevenths (2 fifths down), for example, are altered about 20/26, 15/26, and 10/26 of the way from Pythagorean to 7-based ratios — leaving then respectively about 6/26, 11/26, and 16/26 of the comma from these ratios of 7:9, 6:7, and 4:7. Dispersing a 27.26-cent comma over a chain of only two, three, or four fifths is a somewhat heroic labor, inevitably involving appreciable compromises.

E4	-- +64 --	F4	D4	-- -214 --	C4
(214)		(493)	(278)		(0)
D4	-- -214 --	C4	B3	-- +64 --	C4
(493,278)		(493,0)	(707,429)		(707,707)
B3	-- +64 --	C4	G3	-- -214 --	F3
(921,707,429)		(1200,707,707)	(985,707,278)		(707,707,0)
G3	-- -214 --	F3	E3	-- +64 --	F3

(M6-8 + M3-5 + m3-1 + M2-4)   (m7-5 + m3-1 + M3-5 + m3-1)

In these closest approach progressions, each unstable interval resolves by a total expansion or contraction of 278 cents, the size of the near-6:7 minor third, with the voices moving by 214-cent whole-tones and incisive 64-cent semitones.

These last two cadences, in connection with the previous three-voice example, demonstrate another facet of dispersing the Archytan comma. The 214-cent whole-tone can represent a just ratio of either 8:9 (e.g. C4-D4 in the opening G3-C4-D4 of the three-voice example, 0-493-707 cents as an approximate 6:8:9) or 7:8 (e.g. D4-E4 in G3-B3-D4-E4 of the first four-voice example, 0-429-707-921 cents as an approximate 14:18:21:24).

To sum up, this variety of temperament shares many neo-medieval traits with just or near-just counterparts observing the comma (e.g. efficient closest approach resolutions and compact diatonic semitones), while making it possible seamlessly to combine approximate ratios of 3 and 7 in a simple diatonic scale. Such a system, like a Renaissance meantone for approximate ratios of 3 and 5, presents another side of the tuning equation.

While a chain of fifths at 707.220 cents is of special interest here as a premier element in the larger architecture of 17-WT, this pleasant shade of temperament could also serve as a regular tuning of 12, 17, or 24 notes, for example. If carried to 56 notes, it would nicely circulate.[34] As illustrated in our 17-WT subset, such a tuning would offer augmented seconds at 364.98 cents (Ab-B) and augmented seconds at 342.24 cents (e.g. B-Eb), the former close to 17:21 and the latter to 32:39.

If we were to seek a smaller regular circulating system of this type with a "kinder and gentler" temperament of the fifth than 22-ET, one attractive choice would be 39-ET, with fifths at about 707.692 cents (~5.737 cents wide) or 4/19-Archytas.[35]

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[34] In a 56-note cycle, the last "odd" fifth would be about 4.32 cents narrower than the others, at a near-just size of about 702.90 cents (~0.945 cents wide of a pure 2:3).

[35] For an outstanding near-just circulating system for ratios of 3 and 7 where the Archytan comma is observed rather than dispersed, we might choose 36-ET, with all intervals of 12:14:18:21 (0-266.67-700-966.67 cents) or 14:18:21:24 (0-433.33-700-933.33 cents) within 2.16 cents of just; this tuning also offers a fine approximation of 14:17:21

Fifths are compromised slightly more than in the nearer portion of 17-WT, while many near-7 intervals are slightly more accurate.[36]

However, Secor's 17-WT ingeniously uses an *unequal* temperament to achieve a circulating system that closes in only 17 notes, making the attractions of the 707.220-cent tuning one captivating aspect of a larger design offering a wealth of ratios, interval gradations, and intonational colors. Having placed this system in a neo-medieval perspective, we may now consider its subtle union of elements and some of the musical directions in which it can lead.

(0-333.33-700 cents). Busoni's thirdtone of 66.67 cents is a superb semitone. From a neo-medieval perspective, a main reservation might be that the fifths are gently tempered in the narrow direction (~1.955 cents, as in 12-ET), so that the regular equivalents of Pythagorean intervals are somewhat "diluted," and the usual 100-cent semitones rather wider than optimal. However, I have much enjoyed experimenting with this tuning in a 24-note version with two manuals a sixthtone of 33.33 cents apart.

[36]A charming feature of this tuning is that it offers thirds of the 14:17:21 variety much like those found in temperaments at around 704 cents (Sections 1.2, 1.4), but with a mirrorlike "role-reversal": augmented seconds (~369.23 cents) are larger than diminished fourths (~338.46 cents). This change occurs when one crosses the demarcation line of 17-ET (where the two types of intervals have identical sizes); compare likewise the larger Ab-B with the smaller B-Eb or E-Ab in our subset of 17-WT. The 39-ET diatonic semitone, at about 61.538 cents, is slightly smaller than 27:28, but still within Secor's optimal range of 70±10 cents.