Fifths are compromised slightly more than in the nearer portion of 17-WT, while many near-7 intervals are slightly more accurate.[36]

However, Secor's 17-WT ingeniously uses an *unequal* temperament to achieve a circulating system that closes in only 17 notes, making the attractions of the 707.220-cent tuning one captivating aspect of a larger design offering a wealth of ratios, interval gradations, and intonational colors. Having placed this system in a neo-medieval perspective, we may now consider its subtle union of elements and some of the musical directions in which it can lead.However, Secor's 17-WT ingeniously uses an *unequal* temperament to achieve a circulating system that closes in only 17 notes, making the attractions of the 707.220-cent tuning one captivating aspect of a larger design offering a wealth of ratios, interval gradations, and intonational colors. Having placed this system in a neo-medieval perspective, we may now consider its subtle union of elements and some of the musical directions in which it can lead.

2. An overview of 17-WT: Structure and notations

In some alternative history, 17-tone well-temperaments might have evolved from a 17-note Pythagorean tuning (Gb-A#) of a kind described and advocated in early 15th-century Europe by Prosdocimus de Beldemandis (1413) and Ugolino of Orvieto (c. 1425–1440?), with the varied and often fluid 17-tone systems of Near Eastern theory during this same era providing some possible models also.

In a Pythagorean tuning, 17 pure fifths fall short of 10 pure octaves by about 66.76 cents, an interval which in a larger set such as 24 notes makes an excellent semitone of the "thirdtone" variety, about 3.80 cents larger than the 27:28 of Archytas (see Section 1.3).

To make a 17-note tuning circle close, therefore, we must temper the fifths in the wide direction by a total amount of 66.76 cents, an average of about 66.76/17 or 3.93 cents per fifth, or an average fifth size of about 705.88 cents. In 17-ET, all fifths have this identical size.

In an *unequal* 17-note well-temperament, we are free to use a variety of fifth sizes, some larger and some smaller, as long as this *average* size is maintained

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[36]A charming feature of this tuning is that it offers thirds of the 14:17:21 variety much like those found in temperaments at around 704 cents (Sections 1.2, 1.4), but with a mirrorlike "role-reversal": augmented seconds (~369.23 cents) are larger than diminished fourths (~338.46 cents). This change occurs when one crosses the demarcation line of 17-ET (where the two types of intervals have identical sizes); compare likewise the larger Ab-B with the smaller B-Eb or E-Ab in our subset of 17-WT. The 39-ET diatonic semitone, at about 61.538 cents, is slightly smaller than 27:28, but still within Secor's optimal range of 70±10 cents.

and all fifths remain satisfactorily "playable," typically within about 7 or 8 cents of a pure 2:3.[37]

Secor's 17-WT, when viewed from a neo-medieval perspective, presents a kind of union between two regular tuning systems, incorporating some of their respective intervals and at the same time giving rise to new ones.

Nine fifths in the nearer portion of the circle (Ab-B), as we saw in the last section (1.5), are tempered at about 707.220 cents (~5.265 cents wide), dispersing the Archytan comma and neatly combining approximate ratios of 3 and 7 in a simple diatonic scale. These intervals play a central role in shaping the character and color of the tuning.

The remaining eight fifths are tempered at about 704.377 cents (~2.422 cents wide), so as to generate pure 11:14 major thirds or 7:11 minor sixths — a temperament identical to that we surveyed in Section 1.2. These fifths, together with the others, precisely close the 17-note circle.[38]

Like two voices in an ideal counterpoint, each of which could stand on its own, either of these portions of 17-WT could stand in its own right as the basis for a beautiful regular tuning; but in this intonational counterpoint, the union or fusion of two shadings of temperament gives rise to a new architecture of concord and harmony, which we now consider from three viewpoints.

2.1. A three-dimensional perspective

One aspect of 17-WT is the intervals we have already encountered in the regular tunings at 707.220 cents and 704.377 cents which it combines: for example, near-7:9 thirds or just 11:14 thirds, intervals generated from fifths of a single size. A second aspect is the rainbow-like spectrum of interval sizes and colors often resulting when fifths of the two sizes are mixed in different combinations, as with the many shades of neutral thirds. A third aspect, the "palace of ratios" design, combines just or near-just intervals taken from different parts of the circle

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[37] In the 12-note well-temperaments of late 17th–19th century Europe, fifths typically remain within about 7 cents of pure, although a few schemes temper certain fifths by as much as 1/3 of the 23.46-cent Pythagorean comma (about 7.82 cents), or even 1/2 of this comma (about 11.73 cents). In inharmonic timbres, the range of "musically interchangeable" fifths may be much wider, as is illustrated by intervals often tuned around 685 cents or 720 cents, for example, in Balinese or Javanese gamelan music. I have experimented with synthesizer timbres where a fifth at 21:32 (~729.22 cents), a full Archytas' comma or 27.26 cents wider than pure, could seem quite concordant, or even a "fifth" in 13-ET at 8/13 octave (~738.41 cents). A system which circulates under such special conditions, for example a 17-note tuning circle including fifths as wide as 21:32, might be called a "well-timbrement."

[38] If the eight smaller fifths of 17-WT are defined as generating just 11:14 thirds or 7:11 sixths, then the nine larger fifths would have a size of about 707.220452 cents, with the convenient approximation of 707.220 cents remaining accurate to the nearest thousandth of a cent.

to form "isoharmonic" sonorities involving relations between higher partials (e.g. 7:9:11 or 7:9:11:13), with novel possibilities for neo-medieval or other styles.

In an overview of these three dimensions, we may find a keyboard diagram of the complete 17-note tuning helpful:

	66.741		353.610			562.364		771.120		1057.987		
	Db		D#			Gb		Ab		A#		
C		D		E	F		G		A		B	C
0		214.441		428.882	492.780		707.220		921.661		1136.102	1200

	144.856		278.339			640.479		849.232		985.559		
	C#		Eb			F#		G#		Bb		
C		D		E	F		G		A		B	C
0		214.441		428.882	492.780		707.220		921.661		1136.102	1200

Here both keyboards share in common the seven diatonic notes, with the lower manual including the five more "common" accidentals (Eb-G#) of a typical neo-medieval/Renaissance style, and the upper manual the five others.

Focusing on the first aspect or dimension of the tuning, we find that the Ab-B or 707.220-cent portion of the circle provides a set of six major thirds at 428.88 cents, the best approximations of 7:9. Similarly, the 704.377-cent portion of the circle (moving sharpward from B to Ab or flatward from Ab to B) includes five major thirds at a just 11:14 (~417.508 cents), or minor sixths at 7:11. Characteristic intervals of each regular temperament are combined in a single tuning system.

Our second dimension is the rainbow-like spectrum or gradient of interval sizes and shadings resulting when intervals of a given type are generated from chains mixing the two sizes of fifths. For example, in addition to the near-7:9 and just 11:14 major thirds, we have intermediate sizes of about 426.038 cents (Db-F, D-F#); 423.195 cents (Gb-Bb, A-C#); and 420.351 cents (A#/Cb-Eb, E-G#).

The spectrum of middle or neutral thirds ideally illustrates this rainbow-like effect: these intervals are formed from eight fifths down or nine fifths up (e.g. F#-Bb, F-G#). They range from about 342.236 cents (near 32:39) to 364.984 cents (approaching 17:21), forming a subtle spectrum with seven intermediate sizes arranged in graduated steps of about 2.843 cents, the difference between the two sizes of fifths. In a delightful touch of symmetry, the largest 364.984-cent size, at the exact ratio of 121:98, represents either the augmented second of the 707.220-cent temperament (Ab-B) at nine fifths up, or the diminished fourth of the 704.377-cent temperament at eight fifths down (Fx/Ab-B).[39]

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[39] Thus it is possible to define the larger fifth of 17-WT as having the size of precisely $(1936:49)^{1/9}$, where 1936:49 is equal to the augmented second at 121:98 plus five octaves, or about 6364.984 cents, the interval formed by nine fifths up at about 707.220452 cents. In the 704.377-cent temperament with pure 11:14 thirds, this same interval of 121:98 is a diminished fourth equal to an octave less an augmented fifth of 121:196 (~835.016

This gradient or continuum of sizes is especially diverse for intervals generated from longer tuning chains, thus lending themselves to many different combinations of the two fifth sizes, as with neutral thirds, and also neutral or intermediate sevenths (seven fifths up or ten fifths down). The latter intervals range from about 1049.457 cents (e.g. B-Bb, a virtually just 6:11) to 1069.361 cents (e.g. Ab-F#, a near-7:13).

Melodically, this same rainbow or gradient effect produces diatonic semitones ranging from the near-27:28 size of 63.898 cents for the nearest portion of the circle, to 78.115 cents (approaching 21:22), with all of these steps within Secor's optimal range of 70±10 cents. Chromatic semitones vary from 150.543-cent steps of the 11:12 or "3/4-tone" type (e.g. Bb-B, Eb-E), to 130.639-cent steps of a 13:14 or "2/3-tone" type in the far portion of the cycle (B-Db, F#-Ab).

The third aspect of the tuning as a "palace of ratios" manifests itself when just or near-just intervals from different portions of the circle are joined in sonorities approximating ratios such as 7:9:11:13, a good example to illustrate this dimension of the intonational architecture and its design encompassing higher harmonic partials.

To approximate 7:9:11:13, a rounded 0-435-782-1071 cents in just intonation, we might choose the sonority Ab3-C4-D#4-F#4 (about 0-429-782-1069 cents). The interval Ab3-C4, taken from the nearer part of the circle, has all four of the fifths in its chain at the larger size, so as to yield the best approximation of 7:9 (about 6.202 cents narrow). In contrast, Ab3-D#4 at a just 7:11 draws on the far side of the circle, with all four of its fifths at the smaller size (Ab-Db-Gb-A#-D#), following the chain in the flatward or downward direction. The outer interval Ab3-F#4, about 2.341 cents narrow of a just 7:13, is likewise derived from a flatward chain of seven smaller fifths (Ab-Db-Gb-A#-D#-G#-C#-F#).[40]

Our 7:9:11:13 exemplifies what Secor terms an *isoharmonic* sonority, where intervals share equal or similar difference tones, here 2 (7:9, 9:11, 11:13). The way in which just or approximate ratios from different portions of the circle "line up" to form such combinations suggests the design of a palace or cathedral, or possibly of an ancient astronomical observatory.

cents), twice the size of an 11:14 third. This type of symmetry would hold for any 17-note well-temperament with nine contiguous fifths of one size and eight of another.

[40] Considering the other three intervals of this near-7:9:11:13 will reveal some other nuances of the tuning circle. The near-9:11, C4-D#4, draws on a sharpward chain of five larger fifths and four smaller ones (C-G-D-A-E-B-F#-C#-G#-D#), and at ~353.610 cents is wide of 9:11 by exactly the same amount (~6.202 cents) by which Ab3-C4 is narrow of 7:9, making Ab3-D#4 a just 7:11. The near-11:13, D#4-F#4, has a flatward chain of three smaller fifths (D#-G#-C#-F#), and a size of ~286.869 cents, identical to a regular minor third in the 704.377-cent temperament (about 2.422 cents narrow of 11:13, the same amount by which this fifth is tempered wide). The near-9:13, C4-F#4, has a size of ~640.479 cents, or ~3.862 cents wide, and has a chain in the sharpward direction of five larger fifths and one smaller fifth (C-G-D-A-E-B-F#).

Novel from a perspective of medieval or later styles of historical European composition, isoharmonic sonorities like 7:9:11, 9:11:13, and 7:9:11:13 invite some new types of progressions and resolutions in a neo-medieval setting (Section 7), adding yet further to the many congenial features and resources of 17-WT for such a style.

2.2. Degrees and Notations: Pythagorean and Tartini-Couper

In many neo-medieval contexts, a traditional Pythagorean notation of the kind we have been using serves admirably: such a notation is based on the 17-WT chain of fifths (Gb-A#). In either 17-ET or 17-WT, the smallest interval in the system is a diatonic semitone, so that this conventional notation suffices without the need for any additional symbols to show commas or smaller diesis inflections. This approach names the series of 17 diatonic semitone steps as follows within an arbitrarily chosen octave C-C:

C Db C# D Eb D# E F Gb F# G Ab G# A Bb A# B C

As suggested by George Secor, we may use these 17 tuning steps as a measure of interval categories, with a usual diatonic semitone defined as "one degree of 17," or 1°17 for short.[41] In 17-ET, intervals spanning the same number of tuning steps would have identical sizes; in an unequal well-temperament such as 17-WT, such intervals more or less interchangeably represent the same general category, but often with subtly different sizes and shadings.

For example, any interval spanning six degrees of 17 (6°17) interchangeably represents a regular major third, with sizes ranging from 11:14 or about 417.51 cents (e.g. B-D#, F#-A#) to 428.88 cents (e.g. G-B, Eb-G). Likewise 10°17 always represents a perfect fifth; 7°17 a perfect fourth; 4°17 a regular minor third; and so on.

The main complication with this 17-note Pythagorean spelling arises when intervals wrap across the demarcation line between the extreme flat and sharp regions of the circle, a line bridged by the fifth A#-Gb. We may also write this fifth in a more regular spelling as A#-E# or Cb-Gb, relying on the 17-WT equivalencies A#=Cb and Gb=E#.

Equivalencies of this kind (e.g. D#=Fb, G#=Bbb; Db=B#, Ab=Fx, Eb=Cx) can be helpful in seeking regular spellings for such intervals as the major thirds A#-Eb (Cb-Eb or A#-Cx), G#-Db (G#-B# or Bbb-Db), and D#-Ab (D#-Fx or Fb-Ab), all at 6°17; or the minor third Ab-A# (Ab-Cb or Fx-A#) at 4°17; or the whole-tone D#-Gb (D#-E# or Fb-Gb) at 3°17.

When using Pythagorean spellings for such intervals, I will often follow a convention of giving the usual note name (Gb-A#) along with an equivalent

[41] As Secor observes (personal correspondence), this is by analogy with 1°C for "one degree Celsius."

which seems to fit the musical context. For example, here is a transposition of a familiar four-voice cadence of the closest approach variety:

B#3/Db4	C#4
A#3	G#3
Fx3/Ab3	G#3
D#3	C#3

(M6-8 + M3-5 + m3-1 + M2-4)

The spelling of D#3-Fx3-A#3-B#3 for the unstable sonority may help to convey that D#3-B#3/Db4 is a usual major sixth (13°17); D#-Fx3/Ab3 a usual major third (6°17), Fx3/Ab3-A#3 a usual minor third (4°17); and A#3-B#3/Db4 a usual major second (3°17). At the same time, the spelling D#3-Ab3-A#3-Db4 tells directly in a somewhat "tablature-like" manner where to find this sonority on a keyboard layout such as the one I use.

This system of notation nicely fits a neo-medieval style where diatonic whole-tones (3°17) and semitones (1°17) are the routine melodic steps, in contrast to "chromatic semitones" (2°17) representing accidental alterations and open to use in expressive direct chromaticism (see Section 5.1). From a vertical viewpoint, also, the use of conventional sharps and flats may suggest a contrast between "regular" diatonic intervals and "altered" ones:

C#4	D4	Db4	D4
G#3	A3	Ab3	A3
E3	D3	E3	D3

The first cadence has the unstable sonority E3-G#3-C#4, with regular major third and sixth (6°17, 13°17); in the second, the spelling E3-Ab3-Db4 calls attention to the "special" nature of the diminished fourth E3-Ab4 and diminished seventh Eb-Db4 (5°17, 12°17). Melodically, the spelling of the latter example also also draws our attention to the direct chromatic steps Ab3-A3 and Db4-D4 (2°17).

As a neo-medieval style moves further from the diatonic modes and accidentalism of 13th-14th century Europe, however, the very strengths of this notation can become its weaknesses. Some medieval Near Eastern scales beautifully realized in 17-WT, for example, have steps of 3°17 and 2°17, and none of 1°17. An apt notation should reflect rather than contradict the musical patterns of such scales and modes, where middle or neutral thirds and sixths (5°12, 12°17) likewise represent routine rather than "special" intervals.

Secor has adapted a "fractional" notation to 17-WT, where it can felicitously express scale structures of this variety by using two additional symbols:

semisharps (≠) and semiflats (d) respectively raising or lowering a note by 1°17.[42] Comparing this system with a Pythagorean one may clarify both notations:

C	Db	C#	D	Eb	D#	E	F	Gb	F#	G	Ab	G#	A	Bb	A#	B	C
C	C≠	Dd	D	D≠	Ed	E	F	F≠	Gd	G	G≠	Ad	A	A≠	Bd	B	C

In either system, a letter name in combination with a single accidental suffices uniquely to identity any of the 17 steps. Many scales lend themselves to a style of notation using fractional symbols for some steps and conventional ones for others, as with George Secor's spelling of this "symmetrical scale" growing out of our correspondence (Section 4.2), shown together with a Pythagorean spelling:

C	Dd	Eb	F	G	Ad	Bb	C
C	C#	Eb	F	G	G#	Bb	C

The notation with the fractional accidentals may better convey that 2°17 steps such as C-Dd-Eb or G-Ad-Bb are routine intervals in this scale — a 17-WT variation, as we would learn in due course, on a diatonic scale described by the scholar Ibn Sina (Latinized as Avicenna) about a millennium earlier.

Sometimes I find myself shifting between fractional and Pythagorean approaches to notation within a single passage as the musical context shifts from a scale such as the above to one more reminiscent of Gothic Europe. This meeting of scales, and notations, illustrates the diversity of 17-WT.

2.3. Degrees of 17 and interval categories

With either a Pythagorean or fractional approach to notation, we can Secor's measure of "degrees of 17" to specify the basic interval categories of the tuning. For example, 5°17 impartially measures a middle or neutral third whether it represents an integral interval of a scale, or a touch of neo-Gothic "accidentalism" in a setting with regular thirds at 6°17 and 4°17.

Given that 17-WT is an unequal well-temperament, with many of these basic categories offering a rich continuum of interval sizes (Section 2.1), the following table seeks only to give some idea of this variation by showing the smallest and largest sizes in cents for each category. Examples use a Pythagorean spelling to show equivalencies such as 5 fifths down (E-F) or 12 fifths up (Ab-G#) for 1°17:

[42] Thus see his original presentation in *Interval* (1978), n. 1 above, where semisharps and semiflats are used to illustrate scales and keys with neutral intervals such as thirds and sixths. Semisharps could be taken as resembling the 16th-century diesis sign shaped rather like an "x" and used in a practical microtonal notation by Fabio Colonna (1618) to show a note raised by a diesis or fifthtone (around 128:125, ~41.06 cents, or 1/31 octave, ~38.71 cents) equal to about half the size of a chromatic semitone on his keyboard in a 31-note cycle of 1/4-comma meantone or 31-ET. However, it was Tartini who presented the semisharp in the form adopted by Ivan Vyschnegradski, who added a semiflat for his quartertone notation later modified by Mildred Couper.

Degrees	*Basic type*	*Fifths up/down*	*Range in cents*	*Examples*
1°17	minor 2nd	5 down/12 up	63.898–78.115	E-F, Ab-G#
2°17	neutral 2nd	7 up/10 down	130.639–150.543	C-C#, F#-Ab
3°17	major 2nd	2 up/15 down	208.754–214.441	G-A, D#-Gb
4°17	minor 3rd	3 down/14 up	278.339–286.689	E-G, Ab-A#
5°17	neutral 3rd	8 down/9 up	342.236–364.984	B-Eb, Gb-A
6°17	major 3rd	4 up/13 down	417.508–428.882	F-A, G#-Db
7°17	4th	1 down/16 up	492.780, 495.623	C-F, Gb-A#
8°17	dim 5th	6 down/11 up	556.677–573.738	B-F, Db-F#
9°17	aug 4th	6 up/11 down	626.262–643.323	E-Bb, G#-Eb
10°17	5th	1 up/16 down	704.377, 707.220	D-A, A#-Gb
11°17	min 6th	4 down/13 up	771.198–782.492	B-G, Eb-A#
12°17	neutral 6th	8 up/9 down	835.016–857.764	C-G#, D#-C
13°17	maj 6th	3 up/14 down	913.131–921.661	E-C#, D#-Db
14°17	min 7th	2 down/15 up	985.559–991.246	A-G, Db-A#
15°17	neutral 7th	10 up/7 down	1049.457–1069.361	Ab-F#, E-Eb
16°17	major 7th	5 up/12 down	1121.885–1136.102	F-E, A#-Bb

An advantage of a category like 15°17 is that it can apply across a wide range of scale structures and musical contexts while minimizing style-specific connotations, whether an interval is spelled as Ab-F#, Ab-Gd, or G≠-F#.