6. Vertical color and cadential action

WITH AN UNEQUAL TEMPERAMENT such as 17-WT, the element of shifting interval sizes and colors is a pervasive one. It routinely manifests itself, for example, in the subtle contrasts between the relatively blending but unstable sonorities in these two versions of a standard progression:

D4	E4	F4	C#4	D#4	E4
D4		C4	C#4		B3
B3		C4	A#3		B3
G3		F3	F#3		E3

The opening sonority of the first version, G3-B3-D4, has a major third of around 428.88 cents which I described as "exquisitely brilliant" and "sunny" in the rather gentle "Puff Pipes" timbre I used for my first explorations of 17-WT[68]; this combination, an approximate 14:18:21, leads to the cadential sixth sonority G3-B3-D4-E4 at a tempered 14:18:21:24, with similar qualities.

The second version, in a more remote portion of the tuning circle, opens with F#3-A#3-C#4, the pure 14:11 major third having for me a more "mellow" quality lending a pleasant element of "modal color"[69]; here the cadential sixth sonority F#3-A#3-C#4-D#4 approximates a just 66:78:99:112 (11:14 major third, 33:56 major sixth).

In a neo-medieval style along 14th-century European lines, using G3-B3-D4 or F#3-A#3-C#4 as a kind of "pregnant pause" at the end of a phrase or section might bring these coloristic qualities to the forefront, with such impressions varying from timbre to timbre and also from listener to listener.

As I further explored 17-WT, and exchanged experiences with George Secor, some other forms of coloristic harmony emerged, and in this section I consider two examples of idioms illustrating both the expressive shades and nuances available, and some musical patterns with which they may interact.

6.1. A sweet moment in 17-WT

In a letter to Secor (10 October 2001), I considered the way in which medieval elements can mix in my style with 20th-century ones:

> For example, consider this 17-WT moment that came to me:

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[68] See the excerpt from my first letter to George Secor (5 September 2001), at p. 2 above; and also my Alternate Tuning List article "Another beautiful Secor tuning -- Note for Heinz Bohlen," http://groups.yahoo.com/group/tuning/message/27862 (4 Septenber 2001).

[69] "Another beautiful Secor tuning," ibid. As I remarked in this Tuning List post, I was also drawn to the pure 11:7 minor sixths (~782.49 cents) in the more remote portion of the circle, which can have for me "a bit of a pleasantly 'languid' flavor."

C4 D4 C4
A3 B3 C4
G3 ------ F3
E3 ------ F3

> This bit of sweetness has a cadence found in one modern reading of a piece by Perotin around 1200, plus those beautiful 64-cent semitones; but the first two chords suggest to me a "minor seventh" kind of feeling from some popular music around the 1950's or 1960's, rich and quite concordant.[70]
>
> Where did I get that opening E3-G3-A3-C4? -- from some of your examples. There's the cross-pollination in action, and another beautiful gift for which I say "Thank you!"[71]

Secor, in turn, suggested some delightful variations (Letter, 23 October 2001):

> In response to your 17-WT moment (above), once again I have retained the third-inversion subminor seventh chord (a good name for 12:14:18:21?) while going for melodic neutral seconds, which gives:
>
> C4 C#4 C4
> A3 A#3 C4
> G3 F#3 F3
> E3 D#3 F3
>
> All four voices resolve by neutral seconds to the open fifth. To avoid the excessive doubling in the final chord (technically a non-chord, since it has only two different notes), the first two chords may be inverted:

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[70] For a transcription of the opening portion of Perotin's great four-voice composition *Sederunt principes* (dating in one suggested chronology to around 1198), with a cadence from E3-G3-B3-D4 to F3-C4, see Richard H. Hoppin, ed., *Anthology of Medieval Music* (New York: W.W. Norton, 1978), pp. 59–68 at 68, mm. 137–138.

[71] The notation of my sweet moment in the above quote follows a convention of using dashed lines to show that the notes in the lower voices E3 and G3 are sustained while the upper voices move. Often, however, I have found it more convenient to use a style where each note is taken to be sustained until another follows, or a rest symbol (for which I use "r"), or the end of the example. This second approach, generally favored in this article, simplifies writing but can lead to amusing results if one fails to indicate an intended rest.

E4 D#4 F4
C4 C#4 C4
A3 A#3 C4
G3 F#3 F3

> Then, if you try only the last two chords of the above, leaving out the alto voice, you should find something strangely familiar embedded therein. Thus you can have (two) "sweet" subminor seventh chords, neutral-second melody (two in succession in the tenor), and a neo-medieval cadence all at once. How's that for fusion?

As Secor remarks, his second progression indeed has "something strangely familiar embedded" within it: the specified voices form a three-voice cadence F#3-A#3-D#4 to F3-C4-F4, adding the element of "neutral-second melody" to a familiar 14th-century close such as G3-B3-E4 to F3-C4-F4 (M6-8 + M3-5).[72] Both of his examples feature the 2deg17 motion in all voices for which he would suggest in December, inspired by Ptolemy and Partch, the felicitous name of "equable" (see Section 3.2).

The dialogue accompanying this exchange of musical examples also illustrates how terminology may sometimes shift in different stylistic contexts.

Secor's excellent term "subminor seventh chord" for 12:14:18:21, or its tempered equivalent in 17-WT, helpfully alerts the reader that the proportions for the 6:7 thirds and the 4:7 seventh, based on ratios of 2-3-7, are distinct from those of the 5:6 minor thirds and the 9:16 or 5:9 minor seventh of 16th-19th century European systems based on ratios of 2-3-5.[73] Further, in contexts such as Secor's impressive "neo-18th-century" tonal progressions presented in his companion article, the greater stability of the *sub*minor triad at 6:7:9 vis-a-vis the *super*major triad at 14:18:21 contrasts with the historical 5-limit situation where the 4:5:6 major triad can often be more conclusive than the 10:12:15 minor triad.

In a specifically neo-medieval setting like that of this article, however, I find myself referring to a 17-WT sonority like E3-G3-B3-D4 (0deg17-4deg17-10deg17-14deg17) simply as a "minor seventh" combination. Here the interval sizes nicely fulfill general neo-medieval expectations of usual minor thirds and sevenths somewhere between Pythagorean (27:32, 9:16) and around septimal (6:7, 4:7).

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[72] A Tartini-Couper notation, Gd3-Bd3-Ed4 to F3-C4-F4, may make this affinity to the standard medieval formula even clearer.

[73] Some meantone temperaments have fine representations of both types of sonorities. Even in a basic 12-note version of 1/4-comma meantone with pure 5:4 major thirds, for example, we have both the usual "minor seventh chord" E3-G3-B3-D4 (~0-310-697-1007 cents) and the "subminor seventh chord" Eb4-F#4-Bb4-C#5 (~0-265-697-966 cents); respectively 0deg31-8deg31-18deg31-26deg31 and 0deg31-7deg31-18deg31-25deg31 in a complete 31-note circle of 1/4-comma meantone or the almost identical 31-ET.

Secor's remarks about the closing "nonchord" of my original "sweet moment" in 17-WT, interestingly, could be interpreted in two different ways. In a Gothic or neo-medieval context, this simple fifth F3-C4 can indeed be described as a mere two-voice interval, in comparison with the complete 2:3:4 trine F3-C4-F4 of his second variation. The latter concluding sonority includes an outer 2:1 octave, lower 3:2 fifth, and upper 4:3 fourth, thus achieving what I would term a full consonant "chord" rather than a simple interval.

From Secor's perspective, however, a complete trine might also represent a kind of "nonchord," "since it has only two different notes," here the pitch classes F and C. In a neo-medieval outlook, F3-C4-F4 represents *three* notes, albeit only two pitch classes, with 2:3:4 (or its tempered 17-WT approximation) dividing the outer 1:2 octave harmonically into 2:3 fifth and 3:4 fourth.[74]

As historical background to my "sweet moment," I should explain that sonorities featuring minor thirds and sevenths such as E3-G3-D4 or E3-B3-D4 typically play a rather tense and very effective role in 13th-14th century cadential progressions, but seem to me to take on a more "floating" quality in at least one late 14th-century piece, *Jeo hay en vos* from an English manuscript.[75] In a neo-medieval style, the more "coloristic" aspects of these sonorities often comes to the forefront, influenced by some elements of 20th-century harmony.

In a tuning system like 17-WT, the use of regular minor thirds and sevenths rather close to the simple ratios of 6:7 and 4:7 may tend to accentuate the coloristic and "relatively concordant" aspects of these sonorities, at the same time as it makes traditional closest approach resolutions (m3-1, m7-5) yet more efficient than in the Pythagorean intonation of the Gothic era.

Both my original 17-WT moment and Secor's variations demonstrate an advantage of this type of temperament in which the 64:63 Archytas comma is dispersed: near-septimal sonorities and textures can be effortlessly maintained without the complications of a system observing the comma. Comparing my progression with its equivalents in a tuning with two 12-note Pythagorean sets a 64:63 apart (Section 1.2), or in a 24-note regular tuning with fifths at around 704.61 cents (Section 1.4), may bring out this point; here the signs ^ and * show notes raised respectively by a 64:63 or 27.26-cent comma, or a 55.28-cent diesis:

C4 D4 C4 C4 D4 C4 B*3 C#*4 B*3

---

[74]From a practical as well as theoretical viewpoint, one might say that the 2:1 octave plays a more "essential" role in a 2:3:4 trine, where it is recognized as a kind of "real" interval, than in contexts based on stable sonorities such as 4:5:6 or 6:7:9, where the octave of an arrangement such as 4:5:6:8 or 6:7:9:12 may seem like more of a "mere replication."

[75]For two modern transcriptions, see Richard Rastell, ed., *Four French Songs from an English song-book for voice with 1 and 2 instruments* (North Harton, Lustleigh, Newton Abbot, Devon, England: Antico Edition, 1976), piece 2, pp. 2–3; and Thurston Dart, ed., *Invitation to Medieval Music 1: Music of the earlier fifteenth century* (London: Stainer and Bell, n.d.), piece 6, pp. 8–9.

A3	B3	C4	A^4	B^3	C4	A3	B3	B*3
G3		F3	G3		F3	F#*3		E*3
E3		F3	E^3		F3	E3		E*3
	17-WT			*Pythagorean-based*		*704.61-cent*		

As Secor's equable versions also show, the convenient availability of 2deg17 steps at all locations in the 17-WT gamut facilitates these charming "neutral second" progressions. Again, 17-WT neatly complements the set of just and near-just tunings, each offering its own special qualities.

6.2. Harmonic color and the rebounding seventh

Another kind of coloristic idiom bringing into play the element of unequal temperament in 17-WT is the "rebounding seventh," with the following progression showing a typical rhythmic setting as indicated by the beat numbers and "barlines" above the notes:

1	2 |	1	2	|1 ...	
F4		E4	F#4		G4
D4		E4			D4
B3		C#4			D4
G3		A3			G3

A neo-medieval rebounding seventh sonority consists a major third, fifth, and minor seventh above the lowest note, or 0deg17-6deg17-10deg17-14deg17, with a diminished fifth (here B3-F4) between two of the upper voices. A favorite just tuning is 44:56:66:77, with a pure 11:14 major third, 4:7 minor seventh, and 8:11 diminished fifth (a rounded 0-418-702-969 cents). However, the rebounding seventh can be effective over a range of shadings, of which 17-WT offers a rich assortment including this version at G3-B3-D4-F4, around 0-429-707-985 cents.

Characteristically the outer minor seventh contracts to a fifth, and the upper minor third (6:7 in 44:56:66:77) to a unison, in a standard "closost approach" fashion, while the diminished fifth has a kind of "quasi-resolution" to a mildly unstable minor third (here B3-F4 to C#4-E4), a progression likewise involving a total contraction of a minor third or 4deg17 (8deg17-4deg17). This brings us to the "splitdeg fifth" sonority A3-C#4-E4.

Now the highest voice moves to the major sixth, forming a cadential sonority of A3-C#4-E4-F#4 which expands in a usual resolution to G3-D4-G4, returning to the same note in the lowest voice as the original seventh sonority. The term "rebounding seventh" derives from this pattern, or from the characteristic motion of the outer voices which contract from minor seventh to fifth, and then "bounce back" through the major sixth to the octave of the resolving trine.

A special intonational factor in this 17-WT transposition is the tuning of the middle split fifth sonority A3-C#4-E4 at about 0-423.195-707.220 cents, with a major third very close to 17-ET (~423.53 cents) or 18:23 (~424.36 cents). This may be around

the region of maximum harmonic complexity for a major third, lending an extra degree of dynamic tension or "bounce" to impel the following expansion to a major sixth sonority and trine.[76] In many harmonic timbres, this 423-cent major third can have an especially plangent flavor, while in some gentler timbres it may convey an impression of intriguing richness.

Here is a closely related form of the rebounding seventh progression in a different 17-WT transposition:

1	2 |	1	2	|1 ...
A4		G4	A4	B4
F#4		G4		F#4
D#4		E4		F#4
B3		C3		B3

This time the rebounding seventh moves to the split fifth sonority C4-E4-G4 with the lowest voice of the outer minor seventh B3-A4 ascending by a semitone, and the highest voice descending by a whole-tone, to reach the fifth of this sonority. The following major sixth sonority C3-E4-G4-A4 then resolves to the trine B3-F#4-B4 in a remissive manner, with the lowest voice descending by a semitone and the highest voice ascending by a whole-tone. These motions are the converse of those in the first progression.

Here the rebounding seventh B3-D#4-F#4-A4 has a pure 11:14 major third, with a tuning of around 0-417.508-704.377-985.559 cents, the outer minor seventh remaining as in the last example at the best 17-WT approximation of 4:7. The diminished fifth, equal to the difference of the minor seventh and major third, is therefore rather larger -- about 568.051 cents -- than in the previous example (~556.677 cents) where it is closer to 8:11 (~551.32 cents).

In this second example the split fifth C4-E4-G4 represents the best 17-WT approximation of 14:18:21, taking on a less complex and more "septimal" color than A3-C#4-E4 in the first example. As one moves about the tuning circle, these rebounding seventh progressions -- and related ones involving equable resolutions -- offer many subtle shadings of thirds, sevenths, and other intervals.

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[76] Paul Erlich has found that his “harmonic entropy” model, under one set of assumptions, suggests a region of maximum “entropy” or complexity for the major third at around 423 cents, see http://www.ixpres.com/interval/td/entropy.htm. Dave Keenan has proposed a “Noble Mediant” function for estimating the approximate region of maximum complexity between two simple ratios such as 4:5 and 7:9 for a major third, see Margo Schulter and David Keenan, “The Golden Mediant: Complex ratios and metastable musical intervals,” Alternate Tuning List (17 September 2000), http://groups.yahoo.com/group/tuning/message/12915. The Noble Mediant — a name preferred by Keenan to his originally proposed “Golden Mediant” — is a mediant weighted toward the more complex of the two ratios by a factor of Phi or the Golden Ratio (~1.618034), here (5 + 9 Phi)/(4 + 7 Phi) or ~422.487 cents.