5. Marchettan chromaticism and some "enharmonic" possibilities

EARLY IN THE YEAR 2001, I wrote a paper on the intonational theory and chromaticism of the Italian composer Marchettus of Padua as expounded in his *Lucidarium* (1318)[57], proposing some 24-note keyboard tunings which might approximate his system for singers. Some months later, after I had learned of 17-WT and established contact with its author, George Secor and I noted how nicely this more compact system seems to fit the cadential outlook of Marchettus.

Addressing the intonation of singers rather than the tuning of fixed pitch instruments such as keyboards, Marchettus describes the use of a special cadential semitone or diesis narrower than the usual diatonic semitone of Pythagorean semitone, with unstable intervals such as major thirds expanding to fifths, major sixths to octaves, or major tenths to twelfths consequently made wider than Pythagorean so that they "more closely approach" their stable goal.[58]

Only in the course of writing the present article did I learn that a decade and more earlier, a renowned scholar and interpreter of medieval music was already putting the theory of Marchettus into action: Christopher Page. Taking Pythagorean intonation as a general "starting point" for 13th–14th century French and Italian music, Page finds that many passages "gain incisiveness in performance from wide thirds and correspondingly narrow semitone steps."[59]

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[57] Margo Schulter, *Xenharmonic Excursion to Padua, 1318: Marchettus, the cadential diesis, and neo-Gothic tunings* (2000), available as archive file with ASCII text and PostScript versions at http://value.net/~mschulter/marchetmf.zip, or as ASCII text at http://value.net/~mschulter/marchetmf.txt.

[58] See ibid., and, e.g., Jan W. Herlinger, "Marchetto's Division of the Whole Tone," *Journal of the American Musical Society* 34(2):193-216 (1981); David Lenson, *Nonspecific Accidentals: A study in medieval temperament based on notation* (MA thesis, University of Western Ontario, 1987), Chapter 2; Joseph L. Monzo, *Speculations on Marchetto of Padua's "Fifth-Tones"* (1998), http://www.ixpres.com/interval/monzo/marchet.htm; and Jay Rahn, "Practical Aspects of Marchetto's Tuning," *Music Theory Online* 4.6 (1998), http://boethius.music.ucsb.edu/mto/issues/mto.98.4.6/mto.98.4.6.rahn.html. Marchettus follows the usual pure Pythagorean proportions for the 2:1, 3:2 fifth, 4:3 fourth, and 9:8 whole-tone, but advocates an innovative division of the 9:8 tone into "five parts." The usual limma or *mi-fa* semitone (e.g. E-F, A-Bb) has "two parts," and the usual apotome (e.g. Bb-B) "three parts." In directed resolutions involving accidental alterations such as E3-G#3 to D3-A3 (M3-5) or E3-C#4 to D3-D4 (M6-8), however, the inflected steps G#3-A3 and C#4-D4 are extra-narrow "dieses" or semitones equal to only "one of the five parts of a tone." This division could be interpreted as a geometric one into five equal intervals (very closely resembling 29-ET), or as an arithmetic one in the manner of a monochord partition (for example using the string ratios 81:80:78:76:74:72), or as some other unequal division providing a freer conceptual guide for singers.

[59] Christopher Page, "Polyphony before 1400," in Howard Mayer Brown and Stanley Sadie, eds., *Performance Practice: Music Before 1600* (New York: W.W. Norton, 1990). pp. 79–104 at 80–82, and Examples 1-5.

5.1. Marchettan chromaticism in 17-WT

My paper focused mainly on 24-note systems providing a special cadential semitone narrower than the usual diatonic semitone; this criterion implicitly excluded a 17-tone circle, where regular diatonic semitones are the smallest intervals (1°17). More specifically, if the goal is to model as accurately as possible on a keyboard instrument the kind of intonation postulated for Gothic flexible pitch ensembles and realized in performance by Page, then we might strongly prefer a Pythagorean-based system with its pure stable concords to any temperament.[60]

In freer "neo-medieval" settings, however, 17-WT realizes a musically engaging version of the extra-efficient cadences and striking chromatic progressions of Marchettus, with its diatonic semitones near 28:27 and contrasting chromatic semitones near 12:11 (virtually just in three positions), and its major thirds and sixths not far from 9:7 and 12:7.

Sharing this exciting realization with George Secor in a letter of 9 October 2001, I illustrated these points with examples of chromatic progressions from the *Lucidarium*, some of which are also found in 14th-century pieces by Marchettus and other Italian composers. As I remarked: "Especially in certain portions of the tuning circle, 17-WT can ... capture the contrast Marchettus describes in chromatic progressions between a very wide chromatic semitone and a narrow cadential semitone or diesis," equal in his scheme for singers respectively to "one part" and "four parts" of a whole-tone. The following is a two-voice progression from his treatise "transposed for maximum contrast between these steps":

Eb4	--	+150	--	E4	--	+64	--	F4
(707)				(922)				(1200)
Ab3	--	-64	--	G3	--	-214	--	F3
5		-		M6		-		8

[60] This preference reflects the orientation of Marchettus as well as more conventional medieval theorists to pure ratios for the principal intervals (2:1, 3:2, 4:3, 9:8), interpreted by Page to mandate in practice "a very high degree of accuracy" for stable concords, see "Polyphony before 1400," n. 59 above, p. 79; and also the consideration that, as noted by George Secor in his companion article, the concept of temperament seems generally foreign to this period. Both Secor and I are musically drawn to one solution detailed in my paper: a 24-note Pythagorean tuning with the two 12-note manuals a 64:63 apart, with the cadential diesis of Marchettus at 28:27, and wide major thirds and sixths at 9:7 and 12:7. A pure 24-note Pythagorean tuning (two manuals a Pythagorean comma apart) realizes very similar results. For the kind of monochord-based interpretation favored by Rahn (see n. 58), with smaller dieses at around 48 cents and cadential major thirds and sixths at around 450 cents and 948 cents, one approximation is a system with the two manuals a 459:448 (~41.99 cents) apart, an arrangement also yielding some pure supraminor/subminor or "semi-neutral" sonorities at 14:17:21 of interest from a neo-medieval perspective.

In this progression from fifth to major sixth to octave, the upper voice moves Eb4-E4-F4, the wide 150-cent chromatic semitone followed by an incisive 64-cent cadential semitone representing the "diesis" of Marchettus. Here the ratio betweeen the two semitones of 150:64, or about 2.3:1, is very impressive, however closely it might approach the 14th-century practice which Marchettus models in his division of the tone into "five parts."[61]

Marchettus also describes a "feigned color," or deceptive inflection or cadence where an octave is followed by a major sixth, the upper voice descending by a narrow cadential semitone, with an expected expansion back to the octave, but an actual resolution contracting to a fifth:

F4	--	-64	--	E4	--	-150	--	Eb4
(1200)				(922)				(1200)
F3	--	+214	--	G3	--	+64	--	Ab3
8		-		M6		-		8

In 17-WT, this progression has intervals identical to the last example, but in "time-reversed" order, with the disparity between the two semitones again making possible a musically compelling rendition of Marchettan chromaticism. This effect is especially accentuated, as in these examples, in the nearer portion of the tuning circle at locations where the diatonic semitone (1°17) is smallest and the chromatic semitone (2°17) is largest.

Here are typical neo-medieval versions of these progression in three voices, with the added middle voice bringing into play the melodic interval of an augmented second, or 3°17 in 17-WT:

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[61] The original version in Marchettus is F3-C4 to E3-C#4 to D3-D4, illustrating some fine points of his intonational and notational system. In this system, an inflected progression such as C4-C#4-D4 in the upper voice of our present example is marked by a sign corresponding to our sharp showing an altered step outside of the usual *musica recta* gamut (seven diatonic notes plus Bb), and indicating an extra-large *chroma* C4-C#4 and extra-narrow cadential diesis C#4-D4, the latter appreciably smaller than the usual semitone F3-E3 in the lower voice. In the reading of an equal fivefold division of the 9:8 tone (~203.91 cents), the usual diatonic and chromatic semitones (e.g. A-Bb, Bb-B) would have sizes of around 81.56 and 122.34 cents, with the diesis and *chroma* at around 40.78 and 163.12 cents; 29-ET closely approximates this type of interpretation. A different type of reading draws on a division of the tone in some 14th-15th century theory, taking the "five parts" as more or less equivalent to the Pythagorean comma (531441:524288, ~23.46 cents) and four larger dieses each equal to roughly half of a 256:243 diatonic semitone at ~90.22 cents, or about 45–48 cents. Thus usual diatonic and chromatic semitones would be at or very close to their Pythagorean sizes, with a cadential diesis of ~45-48 cents and a *chroma* or ~156-159 cents. A freer and more moderate interpretation, which both Secor and I find musically appealing, makes the cadential diesis about 28:27 (~62.96 cents) and the *chroma* about 243:224 (~140.95 cents), or respectively about a comma of Archytas (64:63, ~27.26 cents) narrower or wider than the usual Pythagorean diatonic and chromatic semitones.

Eb4	E4	F4	F4	E4	Eb4
Ab3	B3	C4	C4	B3	Ab4
Ab3	G3	F3	F3	G3	Ab4

As I concluded in my letter to George Secor:

> Getting this kind of contrast in a circulating system with only 17 notes is really something! In fact, I'd say that for a style in the direction Marchettus seems to be leading, 17-WT is a superb solution. (Letter, 9 October 2001)

Of course, as Secor observes in his companion article, 17-WT is meant to complement rather than replace just and near-just tunings: the latter approach can provide the pure fifths and fourths and distinctions between semitones appropriate for a more fastidious historical interpretation of the system of Marchettus. However, among its many other attractions, 17-WT vividly evokes many musical qualities of Marchettan cadences and chromaticism with a most admirable economy of means.

5.2. Augmented sixth sonorities

In his initial letter of our correspondence, Secor described a resolution he had tried and liked on the Scalatron featuring a vertical augmented sixth:

> I was ... taken with the effect of voices separated by an interval of 15 degrees (augmented 6th) moving in contrary motion, each by a single degree, to resolve to an octave.
> (George Secor, personal correspondence, 10 September 2001)

Having experienced what I considered the "mysterious" effect of the augmented sixth in 17-ET, I was drawn to it in 17-WT as well, but at first was mistaken as to the location of the virtually pure 11:6 intervals which I knew were a leading feature of the tuning. To begin, I tried a three-voice progression at a location familiar from my experimentation in 17-ET, with the lower voices expanding from major third to fifth (6°17-10°17), and the outer voices from augmented sixth to octave (15°17-17°17):

C#4	D4
G3	A3
Eb3	D3

In response, Secor remarked that "the first chord is rather dissonant," and indeed the progression reminds me of an idiom of the great composer Perotin around 1200 where a sonority such as F3-B3-E4 resolves to F3-C4-F4, the upper voices resolving from tritone to fifth and from major seventh to octave above a sustained lowest voice — both unstable intervals regarded as acute discords. Here the outer augmented sixth and upper tritone (9°17) have a kindred effect.

As I realized after reading this comment (Letter, 26 September 2001), my chosen position for this progression may have accentuated the element of tension by tuning the augmented sixth or 15°17 interval at its next to largest size of ~1066.52 cents, not too far from 13:7 (~1071.70 cents). This gave me the idea of using "key color" to seek a somewhat milder version with a virtually just 11:6.

Such a 17-WT neutral seventh at ~1049.457 cents (by comparison with 11:6, ~1049.363 cents) is actually spelled as a diminished octave (B-Bb, E-Eb, A-Ab), as in this version of the last progression moved to the other side of the tuning circle, where the upper 9°17 interval G#3-Eb4 is spelled as a diminished sixth, equivalent to a tritone in a 17-note circle:

Eb4 D#4
G#3 A#3
E3 D#3

The effect of neo-medieval progressions along these lines, at least for me, is to suggest a kind of fusion between the Gothic of the 13th-14th centuries and the "Gothic" of 19th-century Romanticism or Impressionism as it might have developed in some alternative history.

As it happens, my "rather dissonant" cadence also inspired George Secor to explore some more elaborate progressions involving the augmented sixth, as recounted in his own article.

In its near-13:7 form, the 15°17 interval can participate in what Secor terms an "isoharmonic" sonority of 7:9:11:13 (best approximation Ab-C-D#-F#), with identical difference tones between successive ratios (see Section 7.3).

5.3. Diminished seventh and quasi-Helmholtz sonorities

Another sonority typically involving a resolution where two voices move by 1°17 steps is the 17-WT "diminished seventh" with three 4°17 minor thirds forming the outer interval of a 12°17 neutral sixth or diminished seventh (e.g. E3-G3-Bb3-Db4). While the outer interval of an augmented sixth sonority invites an outward resolution to the octave, the diminished seventh lends itself to an inward resolution to the fifth (12°17-10°17):

Db4 C4
Bb3 C4
G3 F3
E3 F3

(d7-5 + m3-1 + m3-5 + m3-1)

This sonority includes two diminished fifths of 8°17, E3-Bb3 and G3-Db4, which resolve by parallel motion to perfect fifths. The lower and upper minor thirds contract to unisons, while the middle one expands to a fifth, for a colorful effect quite different than that of the 18th-century diminished seventh.

Another type of 17-WT sonority occurred to me after Secor, in one of his letters, mentioned to me that Hermann Helmholtz had used the supraminor or small neutral third of 17:14 (~336.13 cents) in a just version of the Classic diminished seventh at 10:12:14:17, a rounded 316-583-919 cents (Personal correspondence, 5 November 2001).

Eventually this gave me the idea for a "quasi-Helmholtz" sonority sharing the elements of an outer interval very close to a just 17:10 (~918.64 cents), here a regular major sixth (13°17); two intervals of the diminished fifth or augmented fourth type (8°17 or 9°17); and a minor third close to 7:6 — or actually two!

As it happens, the major sixth A-F# has a size of ~918.818 cents, a virtually pure 17:10, with the other intervals falling neatly into place — but in a pattern quite distinct from the 19th-century framework of Helmholtz with its system of harmony based on ratios of 5:

F#4 G4
Eb4 D4
C4 D4
A4 G3

(M6-8 + m3-5 + m3-1 + Aug2-4)

While the major sixth in this sonority of 0-4-8-13 steps expands to an octave in usual Gothic fashion, a characteristic touch here is the resolution of the upper augmented second to a fourth (5°17-7°17) with both voices moving by 1°17. In this location with its near-17:10 sixth, the upper 5°17 interval has a size of ~362.141 cents, a bit larger than 16:13 (~359.47 cents) and not too far from 21:17 (~365.83 cents), the fifth's complement of Helmholtz's 17:14, to which it offers a curious analogy. In other locations, of course, the intervallic colors would somewhat vary.

This resolution has a tritonic tartness shared by some progressions of the 13th and 18th centuries: changing the next to highest voice from Eb4 to E4 would produce A3-C4-E4-F#4 to G3-D4-G4, a type of four-voice formula favored around 1200 (e.g. G3-Bb3-D4-E4 to F3-C4-F4) as a bold final or internal cadence. Gothic and Classic associations agreeably meet, with the resolution of the upper neutral third (5°17) to a fourth by contrary motion as a new element in this 17-WT rendition.

5.4. Enharmonic progressions with building tension

Some examples from George Secor of "enharmonic" progressions in four voices (Letter, 26 September 2001), with a sense of tension building toward the cadence, inspired me to look for analogous idioms in a neo-medieval style. We refer to these progressions as "enharmonic," because they involve one or more voices moving by successive steps of 1°17, an interval defining both the diatonic semitone and the enharmonic diesis (e.g. Db-C#) in a 17-note circle.

The following impressive example from that letter may give some idea of the model I sought to emulate. Here the note in the next to highest voice, A3, is sustained while the other three voices progress in enharmonic steps:

C4	C≠4	C#4	D4
A3			
G3	Gd3	Gb3	F3
E3	Ed3	Eb3	D3

As Secor writes, "The dissonance builds until the final resolution to the consonant subminor triad." In this progression, and in some 17-WT cadences adapted from Classic 18th-century harmony which he shared with me in our early correspondence and discusses in his companion article, the tempered 6:7:9 (here D3-F3-A3-D4, approximately 6:7:9:12) serves as the main stable sonority.

To this most dramatic example, he appended a likewise noteworthy remark:

> Somehow I can just imagine you're saying, "Toto, I don't think we're in the Middle Ages anymore!" Nor, for that matter, is this Kansas, or any other time and place we've ever been before. I have a feeling that this new century is going to have many more *jamais vu* experiences for us. (Letter, 26 September 2001)

A more neo-medieval style of 17-WT enharmonicism, with 2:3:4 or its tempered equivalent as the primary unit of concord, would have to follow somewhat different patterns of "harmonic geometry," as I call it, but might likewise lead to Secor's "*jamais vu*," the experience of the "never before seen."

Eventually I found myself coming up with this four-chord sequence, here introduced by an equable cadence setting D4-A4 as the momentary center:

A4	Cb5/A#4	A4	A4	Cb4/A#4	B4	C5
A4	Ab4	A4	A4	Ab4	G4	F4
D4	Fb4/D#4	D4	D4	Fb4/D#4	E4	F4
D4	Db4	D4	D4	Db4	A3	F3

The expansion of the outer voices from fifth to twelfth, and the increasing tension culminating in the penultimate near-4:6:7:9 sonority (A3-E4-G4-B4) with its resolution to a near-1:2:3 (F3-F4-C5), is marked by the enharmonic motions of the second lowest and highest voices: Fb4/D#4-E4-F4 and Cb5/A#4-B4-C5. This sequence reflects my perception of 4:6:7:9, suggested to me by Paul Erlich in the summer of 2000 as a possible neo-medieval sonority[62], as rather more tense than a usual minor seventh sonority such as Db4-Fb4-Ab4-Cb4.

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[62]Erlich's original query concerned the more complex 4:6:7:9:11, nicely represented also as it happens in 17-WT (e.g. A3-E3-G3-B3-Eb4).

While the 4:6:7:9 or its tempered 17-WT equivalent often suggests to me an intriguingly complex relative concord almost verging on discord, a subjective reaction doubtless reflecting my own historical and stylistic background[63], it is a chord of Secor's isoharmonic type, with difference tones between successive voices equal to either 1 or 2. A simpler isoharmonic form would be 6:7:8:9 (represented in 17-WT, for example, by C4-Eb4-F4-G4). These sonorities, although unstable and decidedly tense in a typical neo-medieval setting, can very well, as Secor points out, serve as primary concords in other contexts.[64]

Thus our dialogue on enharmonic progressions may illustrate not only the range of *jamais vu* experiences possible in 17-WT, but the often contextual nature of concord/discord in a range of musical styles and tuning systems.

5.5. Enharmonic "drift," or "sliding" in thirdtones

A gentler or more "drifting" kind of was suggested to me by analogy to a "sliding of the voice" described by musician and theorist Fabio Colonna in 1618 in his treatise on *La Sambuca Lincea*, a harpsichord with 31 notes per octave arranged in a circulating system, likely 1/4-comma meantone with pure 5:4 major thirds, a tuning almost identical to 31-ET with each tone divided into five equal dieses or fifthtones.[65]

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[63] In my four-chord enharmonic progression above, the 4:6:7:9 sonority has a "standard" neo-medieval resolution, here A3-E4-G4-B4 to F3-F4-C5, which could be analyzed from an historical viewpoint as a union of two three-voice Gothic progressions. The lower pair of voices and highest voice move A3-E4-B4 to F3-C5, with the unstable outer major ninth of a relatively concordant 4:6:9 sonority expanding to a stable twelfth (M9-12); the upper three voices progress E4-G4-B4 to F4-C5, a routine 13th-14th century resolution (m3-1 + M3-5) taking on a septimal cast in the nearer portion of the 17-WT circle. In the four-voice version, the directed resolutions by contrary motion are thus (M9-12 + m3-1 + M3-5).

[64] In his companion article, Secor suggests that a pentatonic sonority of 12:14:16:18:21 (realized in 17-WT, for example, as C4-Eb4-F4-G4-Bb4) might be used as a restful closing concord, and notes the included isoharmonic pattern of the first four notes at 6:7:8:9. He suggests that this chord might used either in this "closed position (perhaps as an arpeggio), or with the notes spaced by fourths (similar to the open strings of a guitar)," e.g. G3-C4-F4-Bb4-Eb5.

[65] Fabio Colonna, *La Sambuca Lincea, overo Dell'Istromento Musico Perfetto, con annotazioni critiche manoscritte di Scipione Stella (1618-1622)*, ed. Patrizio Barbieri, Musurgiana 24 (Lucca: Libreria Musicale Italiana, 1991), ISSN 1121-0508, ISBN 88-7096-026-9. This fascimile edition includes a very helpful commentary in Italian and English. Colonna's Sambuca Lincea, or "Lynxian Sambuca," with a triangular shape suggesting an ancient Greek harp known as a sambuca, and likely named in honor of the *Accademia dei Lincei* or Academy of Lynxes of which Galileo Galilei was also a member, featured a keyboard with six ranks of seven-note diatonic octaves arranged in ascending fifthtones, and thus some steps of the 31-note cycle are available in more than one location for easier playing.

Among Colonna's musical examples are an "example of circulation," a brief piece successively cadencing on all 31 steps as the music circumnavigates the circle of fifths and returns to the opening sonority[66]; and some examples of what he terms *una strisciata di voce*, i.e. "a sliding of the voice" through a series of enharmonic fifthtone steps leading to a cadential resolution. Here I use an asterisk (*) to show a note raised by a fifth of a tone, with slightly differing sizes for these steps of 128:125 (~41.06 cents) and about 34.99 cents in 1/4-comma meantone, and a uniform size of 1/31 octave or about 38.71 cents in 31-ET[67]:

C5	C*5	C#5	Db5	D5
G4				A4
E4				F#4
E3				D4

In this remarkable passage, the "sliding" upper voice passes through four different types of sixths above the bass (21°31, 22°31, 23°31, 24°31), as well as such a novel interval in an early 17th-century setting as the superfourth G4-C*5 at about 544.48 cents in 1/4-comma meantone, or about midway between 15:11 (~536.95 cents) and 11:8 (~551.32 cents).

Seeking to translate this idiom into 17-WT, where a whole-tone is divided into three parts instead of five, I arrived at pleasant and rather gentle passages such as this:

C4	Db4	C#4	D4
G3	Ab3	G#3	A3
E3			D3

Here the two upper voices ascend together in parallel fourths, moving from minor to neutral to major forms of thirds and sixths with the lowest voice, and leading to a usual intensive cadence (M6-8 + M3-5). Another version of this "sliding in thirdtones" has the lower voice descending toward an eventual remissive cadence, the upper voices remaining stationary, with the following example starting with a relatively concordant 8:9:12 sonority:

D4				E4
A3				B3
G3	F#3	F+3	F3	E3

Further experimentation has led me to longer passages where a prevailing texture of "drift" may flow from one cadence to another:

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[66] Ibid., pp. 103-110 (facsimile, last page numbered 100), with modern transcription at LVII-LXII.

[67] Ibid., pp. 101-102 (facsimile), with transcription at LXVI-LXVII.

1	2	1	2	1	2	&	1	2	1	2	1	2	
G4				A4	G4	F4	E4				F4		
E4	D4	Dd3	D4	E4		D4	C4	B3	Bd3	B3	C4		
C4	B3	Bd3	Bb3	A3			A3	G#3	G+3	G3	F3		

Starting on the mildly unstable "split fifth" sonority C4-E4-G4, with the outer fifth divided into relatively blending major and minor third, the progression moves first to a remissive cadence on A3-E4-A4, and then to an intensive cadence on F3-C4-F4. Here the numbered beats and "barlines" above the notes show the rhythm, rather like a tablature.

The scheme of cadences, itself typical of the 14th century, combines with "thirdtone enharmonicism" so as to produce an impression which reminds me in some strange way of Don Carlo Gesualdo and his music around 1600. I would like to see how this passage, dedicated to my mother Rose Donaldson's most recent birthday, could be developed into a longer piece, an adventure to which I look forward.

Another kind of enharmonic counterpoint in 17-WT involves expansion by contrary motion from a minor to a major third before a fifth, or from a minor to a major sixth before an octave (respectively 4°17-6°17-10°17, 11°17-13°17-17°17), as in this example starting with the sonority E3-G3-C4:

C4		C+4	Dd4
G3		G+3	Ad3
F3	E3	Ed3	Dd3

This type of counterpoint is also useful in resolutions of isoharmonic sonorities (Section 7).