Ordinaire 24 archicembalo tuning
Tempering the circle of fifths

In this archicembalo temperament, each circle of fifths in 1024−ed2 almost exactly approximates the interval sizes we would get by tempering the fifths in units of 1/20 Pythagorean comma. In John Brombaugh's system of Temperament Units (TU), where the Pythagorean comma at 531441/524288 or 23.460 cents is equal to 720 TU and the syntonic comma at 81/80 or 21.506 cents to almost precisely 660 TU, a 1024−ed2 step is equal to 35.965 TU, which for many purposes may be rounded to 36 TU or an even 1/20 Pythagorean comma.

The following diagram thus shows the tempering of each 12−note circle in rounded fractions of a Pythagorean comma and TU values, as well as the sizes of the fifths in 1024−ed2 steps to document the exact synthesizer tuning. The two 12−note circles are placed at 36.328 cents apart.

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  +216  +216 −144 −144 −180  −144 −144  −144  −180   −144  −144   +216
 +3/10 +3/10 −1/5 −1/5 −1/4  −1/5 −1/5  −1/5  −1/4   −1/5  −1/5  +3/10
Eb    Bb    F    C    G     D    A     E     B     F#    C#    G#     D#/Eb
  605   605  595  595   594   595  595   595   594    595   595   605
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594 steps of 1024 = 696.094 cents
595 steps of 1024 = 697.266 cents
605 steps of 1024 = 708.984 cents

A curious feature of this scheme for each circle is that we have major thirds ranging in size from 6 at 387.891 cents, or 1.577 cents wide of a pure 5/4 (386.314 cents), to the two widest at 424.219 cents, a virtually just 23/18 (424.364 cents), narrow by only 0.146 cents.

In the full 24−note tuning, the DOG or Diversity of Gradations approach is realized through two main factors: the mixture of meantone and wide fifths within each 12 note circle; and the slightly unequal division of a meantone chromatic semitone at 78.516 cents into two diesis or fifthtone steps at 36.328 cents (the spacing between circles) and 42.188 cents, a difference of 5.859 cents (5 steps of 1024−ed2), or almost precisely 1/4 Pythagorean comma. A third and smaller factor is the limited resolution or granularity of 1024−ed2, which requires a slightly unequal temperament of the meantone fifths in order to obtain the desired average size, and thus another minor source of increased "muttly" variety according well with the DOG ideal.

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November 4, 2013