

7. Isoharmonic sonorities (7:9:11, 9:11:13, 7:9:11:13)

ONE OF THE CARDINAL FEATURES of 17-WT is the availability of what Adriaan Fokker or his translator Leigh Gerdine terms “isoharmonic sonorities”⁷⁷ uniting ratios from different portions of the tuning circle, with identical difference tones between successive notes, as with 7:9:11, 9:11:13, and 7:9:11:13. These sonorities express what I call the “palace of ratios” aspect of the tuning, designed as it is to facilitate a sophisticated use of factors of 11 and 13 in conjunction with 3 and 7.

As our correspondence proceeded, I soon faced a musical question: how might I use these isoharmonic sonorities in a neo-medieval setting? The best approach to finding out seemed some experimentation at the keyboard, which in turn gave opportunity for a bit of “theorizing” about my impressions, and some more experimenting.

Through this kind of process, I found that isoharmonic sonorities may invite direct resolutions to a stable sonority such as complete trine; or more elaborate progressions often featuring “enharmonic counterpoint” (Section 5.4) where two or more voices move in steps of $1^{\circ}17$, the diatonic semitone equivalent to an enharmonic diesis.

7.1. The 7:9:11 sonority

The 7:9:11 combination ($\sim 0-435.08-782.49$ cents in its just form) includes a lower 7:9 major third best approximated in the nearer portion of the 17-WT circle, and an outer 7:11 minor sixth available in just form in the remote portion of the circle. The upper voices form a 9:11 neutral third (~ 347.41 cents), tempered in a 17-WT rendition where the overall structure is $0^{\circ}17-6^{\circ}17-11^{\circ}17$.

If we optimize this sonority in 17-WT from the viewpoint of intervals above the lowest note, then Ab-C-D#/Ed provides a version with 7:9 about 6.20 cents narrow and 7:11 pure; the upper 9:11 is wide by the same amount that 7:9 is narrow. While 17-WT provides virtually just 9:11 thirds at C#-F and D-Gb at around 347.923 cents, only about 0.515 cents wide, using these intervals entails less accurate versions of one or both of the other ratios. Thus in Bb-D-Gb, we have the best representations of 7:9 and 9:11, with the outer 7:11 narrow by about 5.687 cents.

One favorite resolution has the outer voices expand “enharmonically” from minor sixth to major sixth to octave ($11^{\circ}17-13^{\circ}17-17^{\circ}17$):

D#4/Ed4	E4	F4
C4	B3	C4
Ab3	G3	F3

⁷⁷*New Music with 31 Notes* (Bonn: Bad Godesburg; Verlag fuer Systematische Musikwissenschaft, 1975), pp. 79–81); my thanks to George Secor for this reference.

This treatment of 7:9:11 has one of the outer voices progress by two successive steps of $1^\circ 17$ (here D#4/Ed4-E4-F4), and concludes with a familiar 14th-century closest approach cadence (M6-8 + M3-5).

7.2. The 9:11:13 sonority

The 9:11:13 combines a lower 9:11 neutral third with an upper 11:13 minor third to form an outer 9:13 tritone or augmented fourth (~0-347.41-636.62 cents), or in terms of 17-WT, $0^\circ 17$ - $5^\circ 17$ - $9^\circ 17$. An excellent near-just version is available at D-Gb-G# (~0-347.923-634.792 cents) with the 9:11 about 0.515 cents wide, the 11:13 about 2.341 cents narrow, and the 9:13 about 1.825 cents narrow. Another outstanding near-just rendition occurs at G-A#/Bd-C#, with an equally accurate 11:13, a 9:11 about 3.359 cents wide, and a 9:13 about 1.018 cents wide.

This isoharmonic sonority lends itself to a kind of contrapuntal pun, in which the numbers 9:11:13 can also represent the expansion of the outer voices in an enharmonic progression of $9^\circ 17$ - $11^\circ 17$ - $13^\circ 17$ -($17^\circ 17$), here shown with two alternative harmonizations in the middle voice:

C#5/Dd5	D5	Eb5	Fb5/Ed5		C#5/Dd5	D5	Eb5	Fb5/Ed5
A#4/Bd4		Bb4	Cb5/Bd4		A#4/Bd4	Bb4		Cb5/Bd4
G4	F#4	Gb4	Fb4/Ed4	or	G4	F#4	Gb4	Fb4/Ed4

In these progressions, the highest voice moves by three successive steps of $1^\circ 17$, and the middle voice by two such steps; the lowest voice has two $1^\circ 17$ steps followed by a descending $3^\circ 17$ whole-tone step for the final intensive cadence. It would also be possible to have three $1^\circ 17$ steps in the lowest voice, concluding with a remissive cadence:

G#4/Ad4	A4	Bb4	C5		G#4/Ad4	A4	Bb4	C5
Gb4/F#4		F4	G4		Gb4/F#4	F4		G4
D4	C#4	Db4	C4	or	D4	C#4	Db4	C4

Thus the 9:11:13 at once introduces a novel type of isoharmonic sonority to the neo-medieval repertory, and invites a form of enharmonic counterpoint which might be used as a basis for sustained passages or pieces in a style somewhat analogous to the chromatic or enharmonic compositions of the Renaissance.

7.3. The 7:9:11:13 sonority

The 7:9:11:13 (~0-435.08-782.49-1071.70 cents in just intonation) may provide one of the more spectacular demonstrations of the “palace of ratios” design in 17-WT, and also of the benefits of *unequal* temperament.

Thus 17-WT, in comparison to 17-ET, permits a more accurate representation of 7:9 major thirds in the nearer portion of the circle, along with some just 7:11

minor sixths and near-just 11:13 minor thirds and 7:13 augmented sixths in its remote portion; both systems share near-just approximations of 9:13.

With its pattern of $0^\circ 17-6^\circ 17-11^\circ 17-15^\circ 17$, the 7:9:11:13 receives its best overall approximation in 17-WT at Ab-C-D#/Ed-F# (~0-428.88-782.49-1069.36 cents). Here the 7:11 is pure; the 11:13 and 7:13 both about 2.34 cents narrow; and the 7:9 about 6.20 cents wide. All of these intervals have their most accurate forms, but the 9:11 has a size other than its closest 17-WT approximation, curiously one very close to that of 17-ET (respectively about 353.610 cents and 352.941 cents).

This situation with 9:11 is the same as for 7:9:11 (Section 7.1), providing a mild illustration of the xenharmonic lesson that it is not always possible, even in the best designed systems, for the best overall representation of a given just sonority to use the most accurate approximation of each of its intervals. Again, therefore, the 9:11 in this best *overall* fit is wide by about 6.20 cents, the same amount by which the 7:9 is narrow, these intervals together forming a just 7:11.

Including both the 7:9:11 and the 9:11:13 as subsets, the 7:9:11:13 has all the intervals of these sonorities plus an outer 7:13 augmented sixth which might be described as a large neutral or submajor seventh, with the best 17-WT rendition at Ab-C-D#-F#.

The expansion of the outer $15^\circ 17$ interval to an octave with each voice moving by a $1^\circ 17$ step — a progression which Secor mentioned to me in his first letter he had been “taken with” on the Scalatron (Letter of 10 September 2001, see also Section 5.2) — guides a typical neo-medieval resolution:

F#4	G3
D#4	D3
C4	D3
Ab3	G3

In this striking resolution the outer voices move by $1^\circ 17$, the next to lowest voice by $3^\circ 17$, and the next to highest voice by $2^\circ 17$.

Exploring progressions like these in 17-WT led me to an awareness that isoharmonic sonorities may occur in a variety of neo-medieval tuning systems. For example, 29-ET has a version of 11:13:15 (~0-289.21-536.95 cents) in near-pure form (~0-289.66-537.93 cents), with all intervals within one cent of just. In the following 29-ET progression, an asterisk (*) shows a note raised by an enharmonic diesis of $1^\circ 29$, about 41.38 cents:

D*5	D#5	E*5	F5
C5	B4	B*4	C5
A4	G#4	G4	F4

From the 11:13:15 sonority A4-C5-D*5 ($0^\circ 29-7^\circ 29-13^\circ 29$), the voices expand to a penultimate cadential sonority of G4-B*4-E*5 ($0^\circ 29-11^\circ 29-23^\circ 29$) with a large “major third” and “major sixth” at around 455.17 cents and 951.72 cents, very

close to 13:10 (~454.21 cents) and 26:15 (~952.26 cents). In the final cadence, the upper voices ascend by 1°29 dieses while the lowest voice descends by a 5°29 whole-tone. Thus isoharmonic sonorities and associated enharmonic progressions can take many forms over a range of tuning systems.